

Planimetria - mieszanka zadań ciekawych ZI-TK

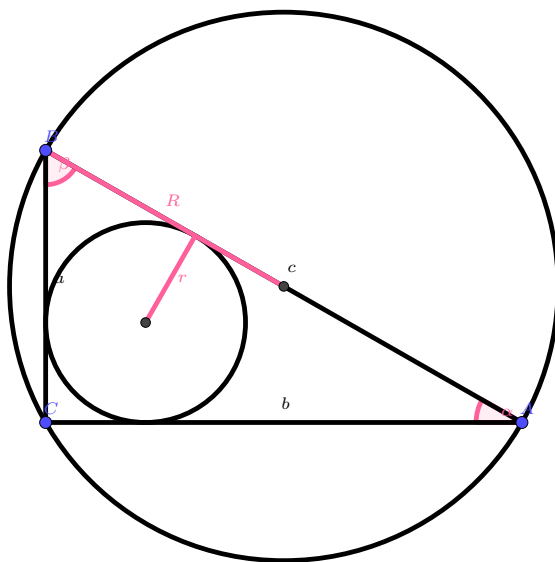
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Treść zadania

Wyznaczyć sinusy kątów ostrych trójkąta prostokątnego wiedząc, że stosunek promieni okręgów opisanego i wpisanego w ten trójkąt jest równy $\frac{5}{2}$.



Rozwiązanie

$$\begin{aligned}\sin\alpha &= \frac{a}{c} & \wedge & \quad \sin\beta = \cos\alpha = \frac{b}{c} \\ \frac{R}{r} &= \frac{5}{2} & r &= \frac{a+b-c}{2} \\ \frac{R}{r} &= \frac{\frac{c}{2}}{\frac{a+b-c}{2}} \\ \frac{R}{r} &= \frac{c}{a+b-c} \quad / : c \\ \frac{R}{r} &= \frac{1}{\frac{a}{c} + \frac{b}{c} - 1} = \frac{5}{2} \\ \frac{5}{2} &= \frac{1}{\sin\alpha + \cos\alpha - 1}\end{aligned}$$

$$\begin{aligned}
5(\sin\alpha + \cos\alpha - 1) &= 2 \\
\sin\alpha + \cos\alpha &= \frac{2}{5} + 1 \\
\cos\alpha &= \frac{7}{5} - \sin\alpha \quad / \quad ()^2 \\
\cos^2\alpha &= \frac{49}{25} - \frac{14}{5}\sin\alpha + \sin^2\alpha \\
1 - \sin^2\alpha &= \frac{49}{25} - \frac{14}{5}\sin\alpha + \sin^2\alpha \\
2\sin^2\alpha - \frac{14}{5}\sin\alpha + \frac{24}{25} &= 0 \quad / \quad : 2 \\
\sin^2\alpha - \frac{7}{5}\sin\alpha + \frac{12}{25} &= 0 \\
\text{niech} \quad t = \sin\alpha \quad \wedge \quad t &\in (0; 1)
\end{aligned}$$

$$t^2 - \frac{7}{5}t + \frac{12}{25} = 0$$

$$\Delta = \frac{49}{25} - \frac{48}{25} = \frac{1}{25} \qquad \sqrt{\Delta} = \frac{1}{5}$$

$$t_1 = \frac{\frac{7}{5} - \frac{1}{5}}{2} = \frac{3}{5} \qquad \vee \qquad t_2 = \frac{\frac{7}{5} + \frac{1}{5}}{2} = \frac{4}{5}$$

$$\begin{aligned}
\sin\alpha &= \frac{3}{5} & \vee & & \sin\alpha &= \frac{4}{5} \\
\sin\beta &= \frac{4}{5} & \vee & & \sin\beta &= \frac{3}{5}
\end{aligned}$$