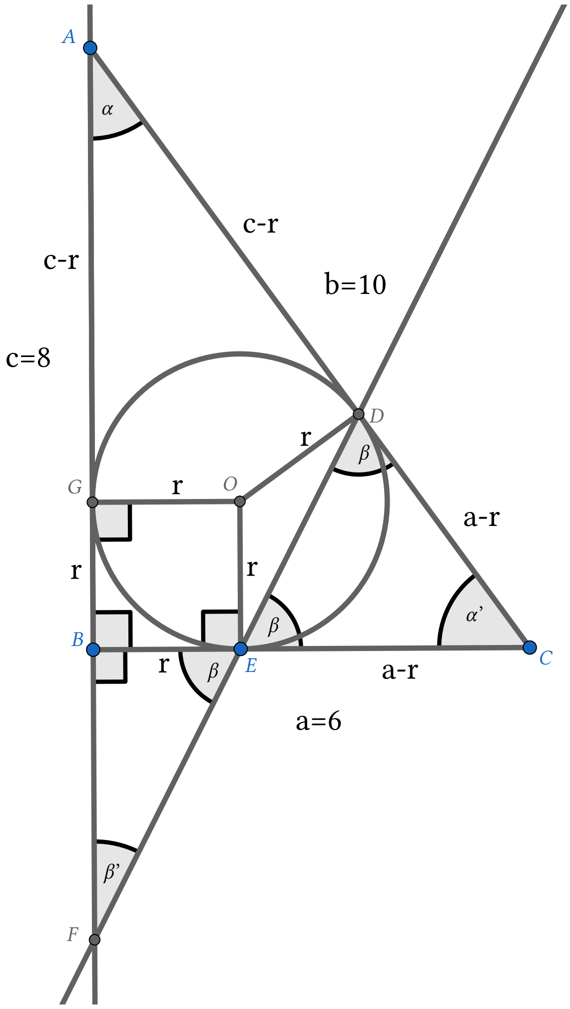
096-PISAREK



$c = |AB| = 8 \quad a = |BC| = 6 \quad b = |AC| = 10$

obliczyć $P_{\triangle EBF}$

$\mathbb{D}\ :\ \alpha, \alpha' \in (0°, 90°)$

$b = a + c - 2r$

$10 = 6 + 8 - 2r \qquad r = 2$

niech $|\measuredangle EDC| = \beta$

$\beta = \frac{180° - (90° - \alpha)}{2} = 45° + \frac{\alpha}{2}$

z $\triangle DEC\ :\ |\measuredangle EDC| = |\measuredangle DEC| = \beta$

z kątów wierzchołkowych: $|\measuredangle BEF| = |\measuredangle DEC| = \beta$

$|\measuredangle FBC| = 90°$, zatem $|\measuredangle BFE| = \beta' = 90° - \beta = 45° - \frac{\alpha}{2} = \frac{\alpha'}{2}$

z $\triangle BEF\ :\ tg\beta' = \frac{r}{|BF|} \qquad |BF| = \frac{r}{tg\beta'}$

$P_{\triangle EBF} = \frac{1}{2}|BE||BF| = \frac{r^2}{2tg\beta'} = \frac{2}{tg\beta'}$

$tg\beta' = tg\frac{\alpha'}{2}$

$$tg\alpha' = \frac{4}{3} = \frac{2tg\frac{\alpha'}{2}}{1-tg^2\frac{\alpha'}{2}}$$

$6tg\frac{\alpha'}{2} = 4 - 4tg^2\frac{\alpha'}{2}$

$2tg^2\frac{\alpha'}{2} + 3tg\frac{\alpha'}{2} - 2 = 0$

$(2tg\frac{\alpha'}{2} - 1)(tg\frac{\alpha'}{2} + 2) = 0$

$tg\frac{\alpha'}{2} = \frac{1}{2} \quad \vee \quad tg\frac{\alpha'}{2} = -2 \notin \mathbb{D} \quad \text{gdyż } \frac{\alpha'}{2} \in (0^\circ, 45^\circ) \Longrightarrow tg\frac{\alpha'}{2} \in (0, 1)$

$tg\beta' = tg\frac{\alpha'}{2} = \frac{1}{2}$

$P_{\triangle EBF} = \frac{2}{tg\beta'} = 4$