

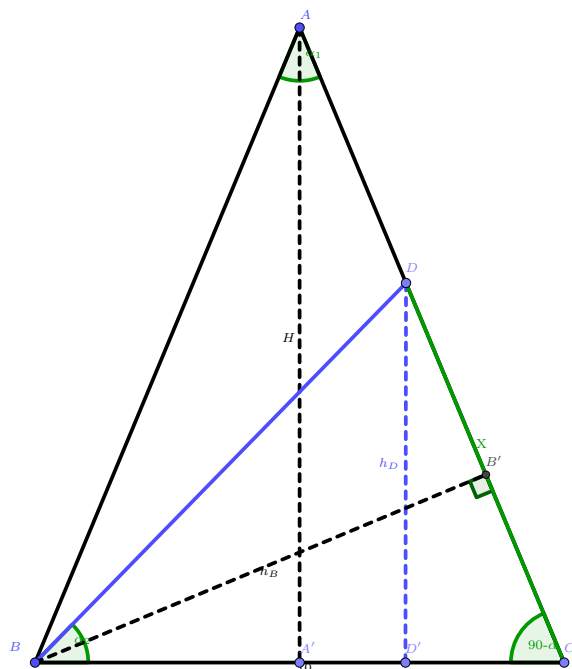
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Treść zadania

Dany jest trójkąt równoramienny ABC, w którym $|AB| = |AC|$ i $|BC| = 10$. Na boku AC wybrano punkt D w ten sposób, że $|\sphericalangle(CBD)| = |\sphericalangle(BAC)| = \alpha$ oraz $|AD| = \frac{69}{13}$. Oblicz $\sin \alpha$.



Rozwiązanie

$$\sin \alpha = \frac{h_D}{BD}$$

$$z \triangleq \overline{BD} \triangle ABC :$$

$$|\sphericalangle ACB| = \frac{180^\circ - \alpha}{2} = 90^\circ - \frac{\alpha}{2}$$

$$z \triangle BCD :$$

$$|\sphericalangle BDC| = 180^\circ - \alpha - (90^\circ - \frac{\alpha}{2}) = 90^\circ - \frac{\alpha}{2} \Rightarrow \triangle BCD \text{ jest równoramienny.}$$

$$|BC| = |BD| = 10$$

$$\begin{array}{l} \text{NIECH} \quad |CD| = x \\ \triangle BCD \sim \triangle ABC \quad (\text{kkk}) \quad \wedge \quad \triangle AA'C \sim \triangle BB'D \quad (\text{kkk}) \end{array}$$

$$\frac{B'D}{A'C} = \frac{BD}{AC}$$

$$\frac{\frac{x}{2}}{5} = \frac{10}{x+\frac{69}{13}}$$

$$x(x+\frac{69}{13}) = 100$$

$$x^2+\frac{69}{13}x-100=0 \quad / \quad *13$$

$$13x^2+69x-1300=0$$

$$\sqrt{\Delta}=\sqrt{69^2+4*13*1300}=\sqrt{72361}=269$$

$$x_1=\frac{-69-269}{26}<0 \quad \vee \quad x_2=\frac{-69+269}{26}=\frac{100}{13}$$

$$|A'C|^2+H^2=|AC|^2$$

$$|AC|=x+\frac{69}{13}=\frac{169}{13}$$

$$H^2=(\frac{169}{13})^2-5^2$$

$$H=\sqrt{\frac{169^2}{13^2}-25}$$

$$H=12$$

$$\frac{AA'}{AC}=\frac{h_D}{CD}$$

$$\frac{12}{\frac{169}{13}}=\frac{h_D}{\frac{100}{13}}$$

$$h_D=\frac{\frac{1200}{13}}{\frac{169}{13}}=\frac{1200}{169}$$

$$\sin\alpha=\frac{\frac{1200}{169}}{10}$$

$$\sin\alpha=\frac{120}{169}$$