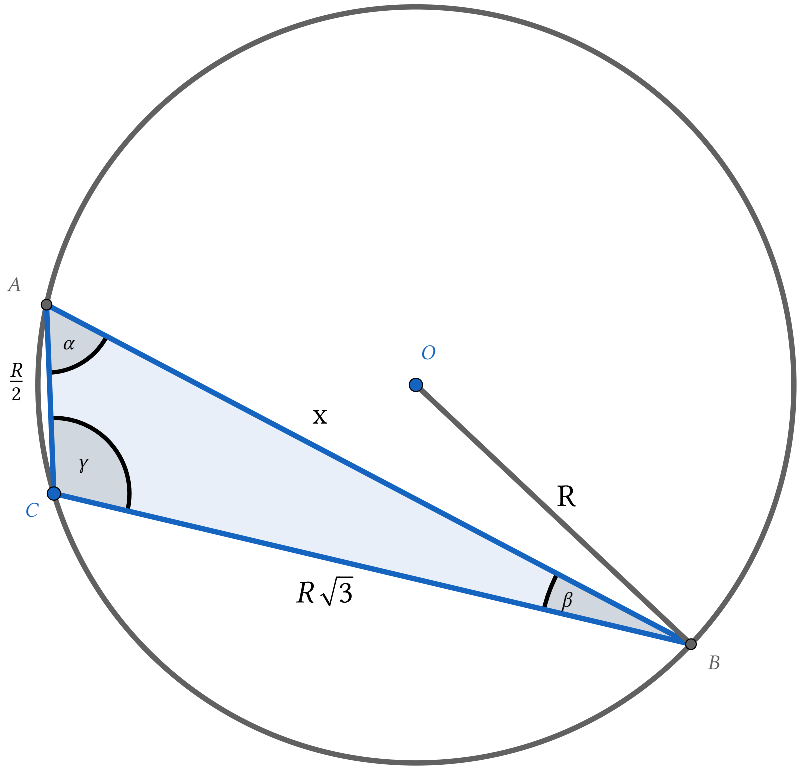
207-PISAREK



$|AC| = \frac{R}{2}$ $\quad |BC| = R\sqrt{3}$ $\quad |OB| = |OA| = |OC| = R$

wyliczyć $x = |AB|$

$\mathbb{D}\ :\ x, R \in \mathbb{R}_{+};\quad \alpha, \beta, \gamma \in (0^\circ, 180^\circ)$

z twierdzenia sinusów dla $\triangle ABC$: $\frac{R}{2sin\beta} = \frac{R\sqrt{3}}{sin\alpha} = \frac{x}{sin\gamma} = 2R$

$\frac{R}{2sin\beta} = 2R \qquad \frac{R\sqrt{3}}{sin\alpha} = 2R \qquad \frac{x}{sin\gamma} = 2R$

$sin\beta = \frac{1}{4} \qquad sin\alpha = \frac{\sqrt{3}}{2} \qquad x = 2Rsin\gamma$

$x = 2Rsin\gamma = 2Rsin(180^\circ - (\alpha + \beta)) = 2Rsin(\alpha + \beta) = 2R(sin\alpha cos\beta + cos\alpha sin\beta)$

$x = 2R(sin\alpha \cdot \pm\sqrt{1 - sin^2\beta} + sin\beta \cdot \pm\sqrt{1 - sin^2\alpha})$

$x = 2R[\frac{\sqrt{3}}{2} \cdot (\pm\sqrt{\frac{15}{16}}) + \frac{1}{4} \cdot (\pm\sqrt{\frac{1}{4}})] = 2R(\pm\frac{3\sqrt{5}}{8} \pm \frac{1}{8}) = 2R(\pm\frac{3\sqrt{5}\pm1}{8})$

$x = \frac{3\sqrt{5}+1}{4}R \quad \vee \quad \frac{3\sqrt{5}-1}{4}R \quad \vee \quad 0 > \frac{-3\sqrt{5}+1}{4}R \notin \mathbb{D} \quad \vee \quad 0 > \frac{-3\sqrt{5}-1}{4}R \notin \mathbb{D}$

$x = \frac{3\sqrt{5}\pm1}{4}R$