$5+2\sqrt{6}-(5-2\sqrt{6})$ $=\underbrace{5+2\sqrt{6}}\,\underbrace{-5+2\sqrt{6}}=4\sqrt{6}$

3.193. 3.211. Doprowadź do najprostszej postaci wyrażenia:

a) $\boxed{\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}}-\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}+\dfrac{1-16\sqrt{6}}{4}$

b) $\dfrac{\sqrt{7}-\sqrt{6}}{\sqrt{7}+\sqrt{6}}-\dfrac{\sqrt{7}+\sqrt{6}}{\sqrt{7}-\sqrt{6}}+\dfrac{8\sqrt{42}+3}{2}$

c) $\dfrac{2}{\sqrt{3}-1}-\dfrac{1}{\sqrt{3}+1}+\dfrac{5-2\sqrt{3}}{4}$

d) $\dfrac{4}{\sqrt{7}-2}-\dfrac{2}{\sqrt{7}+2}+\dfrac{1-2\sqrt{7}}{3}$

$\sqrt{3}\cdot\sqrt{2}=\sqrt{3\cdot 2}$
$\sqrt{6}$

$(a+b)^2=a^2+2ab+b^2$

$$\frac{(\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}=\frac{(\sqrt{3}+\sqrt{2})^2}{3-2}=\frac{3+2\sqrt{3}\cdot\sqrt{2}+2}{1}=5+2\sqrt{6}$$

$(a-b)(a+b)=a^2-b^2$

$$\frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}=\frac{(\sqrt{3}-\sqrt{2})^2}{3-2}=\frac{3-2\sqrt{3}\cdot\sqrt{2}+2}{1}=5-2\sqrt{6}$$

$$4\sqrt{6}+\frac{1-16\sqrt{6}}{4}=\frac{4\sqrt{6}}{1}+\frac{1-16\sqrt{6}}{4}=\frac{16\sqrt{6}}{4}+\frac{1-\cancel{16\sqrt{6}}}{4}=\frac{1}{4}$$

$$\frac{1}{4}-\frac{16\sqrt{6}}{4}$$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a-b)(a+b) = a^2 - b^2$

3.193. **3.211.** Doprowadź do najprostszej postaci wyrażenia:

a) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + \frac{1-16\sqrt{6}}{4}$

b) $\frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}+\sqrt{6}} - \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}-\sqrt{6}} + \frac{8\sqrt{42}+3}{2}$

c) $\frac{2}{\sqrt{3}-1} - \frac{1}{\sqrt{3}+1} + \frac{5-2\sqrt{3}}{4}$

d) $\frac{4}{\sqrt{7}-2} - \frac{2}{\sqrt{7}+2} + \frac{1-2\sqrt{7}}{3}$

$$\frac{4\,(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)} = \frac{4\sqrt{7}+8}{7-2^2} = \frac{4\sqrt{7}+8}{3}$$

$$\boxed{(a-b)(a+b) = a^2-b^2}$$

$$\frac{2\,(\sqrt{7}-2)}{(\sqrt{7}+2)(\sqrt{7}-2)} = \frac{2\sqrt{7}-4}{7-2^2} = \frac{2\sqrt{7}-4}{3}$$

$$\frac{4\sqrt{7}+8}{3} - \frac{2\sqrt{7}-4}{3} + \frac{1-2\sqrt{7}}{3} = \frac{(4\sqrt{7}+8)-(2\sqrt{7}-4)+(1-2\sqrt{7})}{3} =$$

$$\frac{\cancel{4\sqrt{7}}+8-\cancel{2\sqrt{7}}+4+1-\cancel{2\sqrt{7}}}{3} = \frac{13}{3} = 4\frac{1}{3}$$

czwartek, 9 grudnia 2021 18:53

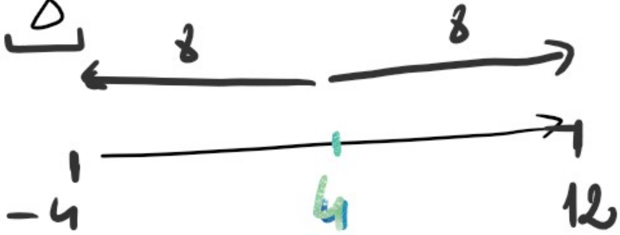
$$\underbrace{|x-4|-8=0}$$

$$|y|=a$$
$$|y|<a$$
$$|y|>a$$

$$\boxed{\begin{array}{l}|x-b|=a\\ |x-b|<a\\ |x-b|>a\end{array}}$$

$$|x-\underbrace{4}|=\underbrace{8}$$

czysty x



$$\boxed{x=-4 \vee x=12}$$

$$4-|5-x|=0 \quad |-4$$

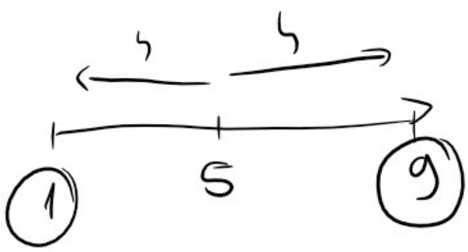
$$-|5-x|=-4 \quad |\cdot(-1)$$

$$|5-x|=4$$

$$\boxed{\begin{array}{l}|a-b|=|b-a|\\ (a-b)^2=(b-a)^2\end{array}}$$

$$|x-5|=4$$

czysty x



odp. $x\in\{1;9\}$

$$\boxed{\begin{array}{l}|-x|=|x|\\ |-a-b|=|a+b|\end{array}}$$

$$8-|-3-x|\geqslant 0$$

$$-|3+x|\geqslant -8 \quad |\cdot(-1)$$

$|3+x|\leqslant 8$

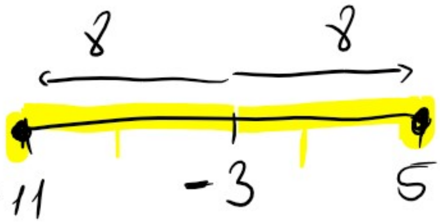
8 8

$|3+x| \le 8$

$|x+3| \le 8$

$x+3=0$

$\boxed{x=-3}$



$x \in \langle -11; 5\rangle$

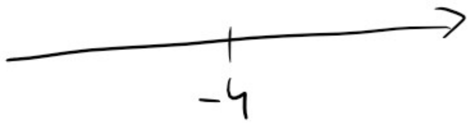
$<$ ostre

$>$ mierne

$\underline{a<b}$

$\underline{a \le b}$

$-5 \le |x+4|$



$|x+4| \ge -5$ záporne

$\boxed{x=-4}$

?

$\ge 0 \quad \ge \quad < 0$

$+ \quad > \quad -$

PRAVDA $\Rightarrow$ $x \in \mathbb{R}$

$|x+4| \ge -5$

$|23+4| \ge -5$

ok

$|-100+4| = |-96| = 96 \ge -5$

$|-4+4| = |0| = 0 \ge -5$

$|-5+4|$

$|-1| = 1 \ge -5$

6 Rozwiąż równanie $\sqrt{16+8x+x^2}-6=0$.

$(a+b)^2=a^2+2ab+b^2$

$(4+x)^2=16+8x+x^2$

$\sqrt{16+8x+x^2}-6=0$

$\sqrt{(4+x)^2}-6=0$

$|4+x|-6=0 \;/+6$

$4+x=6 \;/-4$

$|4+x|=6$

$|x+4|=6$

$-10 \quad -4 \quad 2$

$x=2 \vee x=-10 \iff x\in\{-10;2\}$

7 Wskaż zdanie prawdziwe.

A. Równanie $|x|=-3$ ma dokładnie jedno rozwiązanie.

B. Równanie $1-|x|=0$ ma dokładnie jedno rozwiązanie. 2 rozw.

C. Równanie $|3+x|+1=0$ nie ma rozwiązania. +

D. Równanie $|x-\sqrt{2}|=\sqrt{2}$ nie ma rozwiązania. F

$|x|\geq 0$

0/1

$1-|x|=0 \;|-1$

$-|x|=-1 \;|\cdot(-1)$

$|x|=1 \quad x\in\{-1;1\}$

$-1 \quad 0 \quad 1$

$|x|=0$ 1 rozwiąz.

$|x|=5$ 2 rozw.

$|3+x|+1=0 \;|-1$

$\underbrace{|x+3|}_{\geq 0}=\underbrace{-1}_{<0}$

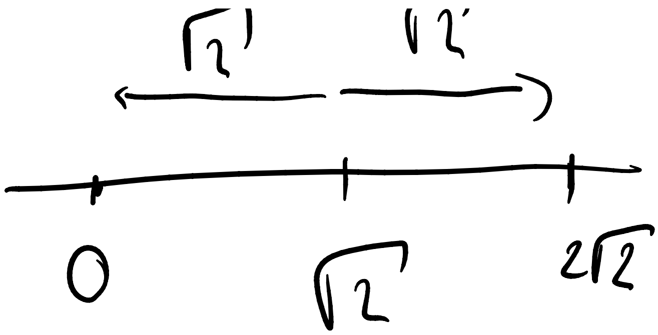
$x\in\varnothing$

$|x-\sqrt{2}|=\sqrt{2}$

$\sqrt{2} \quad \sqrt{2}$

$$|\underbrace{x-\sqrt{2}}| = \sqrt{2}$$

$$\boxed{x = \sqrt{2}}$$



$$|\underbrace{\sqrt{2}-\sqrt{2}}_{0}|$$

$$|2\sqrt{2}-\sqrt{2}| = |\sqrt{2}+\underbrace{\sqrt{2}-\sqrt{2}}_{0}| = \sqrt{2}$$

czwartek, 9 grudnia 2021 19:26

$$\sqrt{9x^2-12x+4}-8\geq 2$$

$$\sqrt{\underbrace{9x^2}_{a^2}-12x+\underbrace{4}_{b^2}}\geq 10$$

$$\sqrt{(3x-2)^2}$$

$$9x^2-\underline{2\cdot 3x\cdot 2}+2^2=\underline{9x^2-12x+4}$$

$$\boxed{\sqrt{a^2}=|a|}$$

$a\in\mathbb{R}$

$\sqrt{a}^2=a$ tylko dla $a\geq 0$

$a\geq 0$

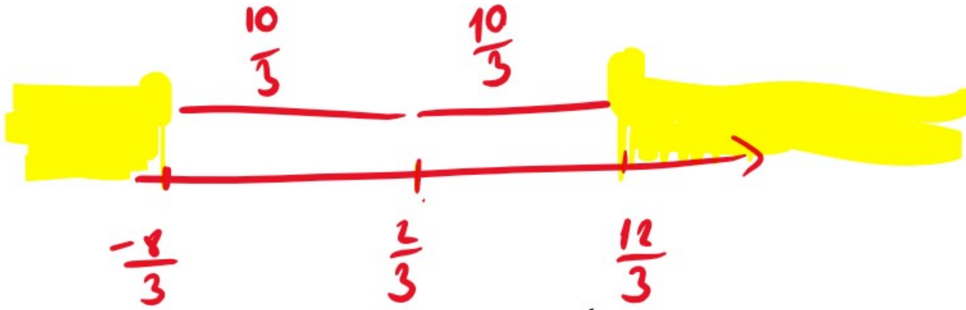
$$|3x-2|\geq 10 \quad |:3$$

TAK!

$3x$ FUJ!

$$\left|x-\frac{2}{3}\right|\geq\frac{10}{3}$$

ZNAK!



$$x\in\left(-\infty,-\frac{8}{3}\right\rangle\cup\langle 4,+\infty)$$

czwartek, 9 grudnia 2021 19:35

$k \in \mathbb{Z}$

$$x = (7k+1)+(7k+2)+(7k+3)+(7k+4)+(7k+5)+(7k+6) =$$

$$6 \cdot 7k + 21 = 2 \cdot \underbrace{3 \cdot 7k} + \underbrace{21} = 21(\underbrace{2k+1}_{\in \mathbb{Z}}) \Rightarrow 21 \mid x$$

czwartek, 9 grudnia 2021 19:39

3.155. Wykaż, że jeśli liczby całkowite *a*, *b*, *c* przy podzieleniu przez 4 dają odpowiednio reszty 1, 2, 3, to reszta z dzielenia sumy kwadratów tych liczb przez 4 jest równa 2.

$y, x, l, k \in \mathbb{Z}$

Z: $a = \underbrace{4 \cdot k}_{\text{podz. p.}} + \underbrace{1}_{\text{reszta}}$, $b = 4l + 2$, $c = 4x + 3$

T: $\underbrace{a^2 + b^2 + c^2} = 4y + 2$

D: $a^2 + b^2 + c^2 = (4k+1)^2 + (4l+2)^2 + (4x+3)^2 =$

$16k^2 + 8k + 1 + 16l^2 + 16l + 4 + 16x^2 + 24x + 9 = 4(4k^2 + 2k + 4l^2 + 4l + 1 + 4x^2 + 6x) + 10$

$10 = 2 \cdot 4 + 2$

$4(\underbrace{(\quad) + 2}_{\in \mathbb{Z} = y}) + 2 \Rightarrow$ reszta $= 2$