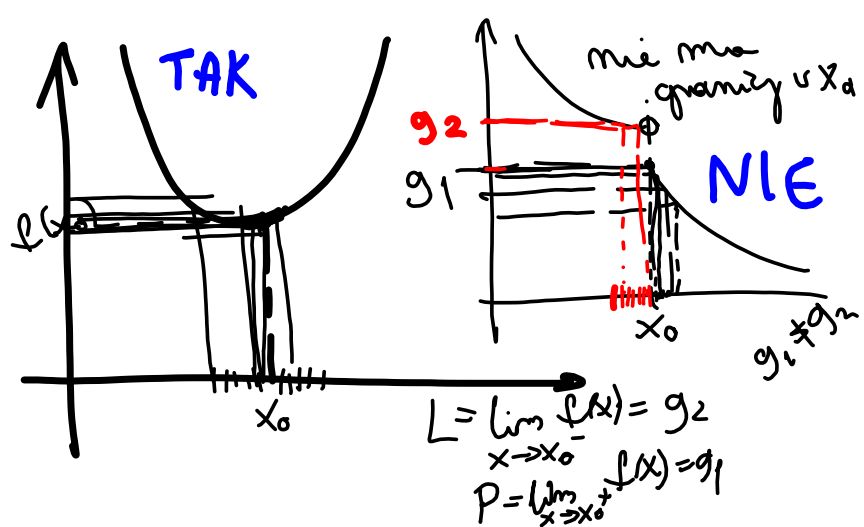
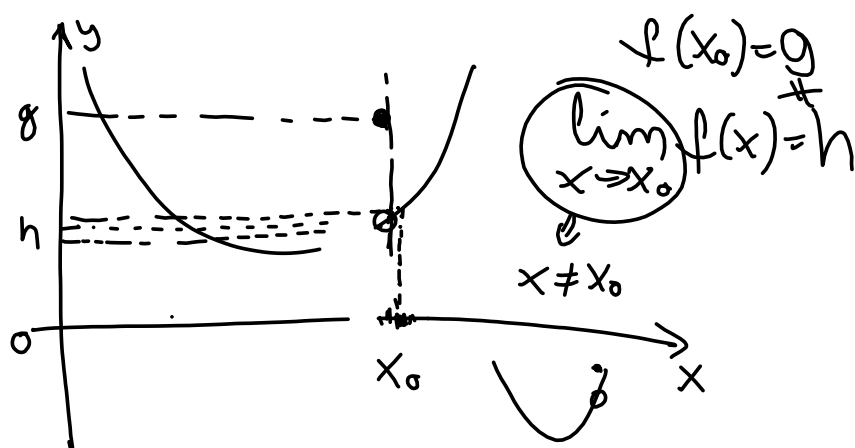


$$f: (a, b) \rightarrow \mathbb{R} \quad \text{cg } x_0 \in (a, b)$$

$$\Leftrightarrow 1) \exists \lim_{x \rightarrow x_0} f(x)$$

$$2) \lim_{x \rightarrow x_0} f(x) = f(x_0)$$

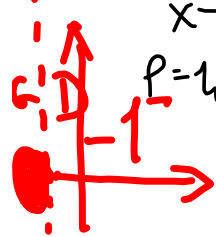




$$F(x) = \begin{cases} x^3 - 2x^2 + 5x - 1 & \text{dla } x \in (-\infty, -1) \\ 4x^2 + 10x - 2 & \text{dla } x \in (-1, +\infty) \end{cases}$$

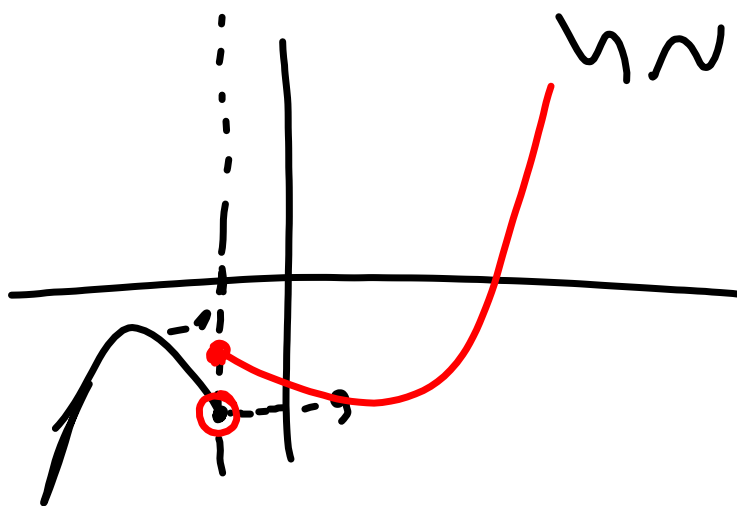
F.d.)

$$L = \lim_{x \rightarrow -1^-} (x^3 - 2x^2 + 5x - 1) = -9$$



$$P = \lim_{x \rightarrow -1^+} (4x^2 + 10x - 2) = -8$$

$$L \neq P \Rightarrow \text{nieciąg}$$

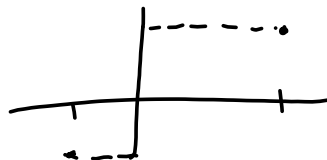


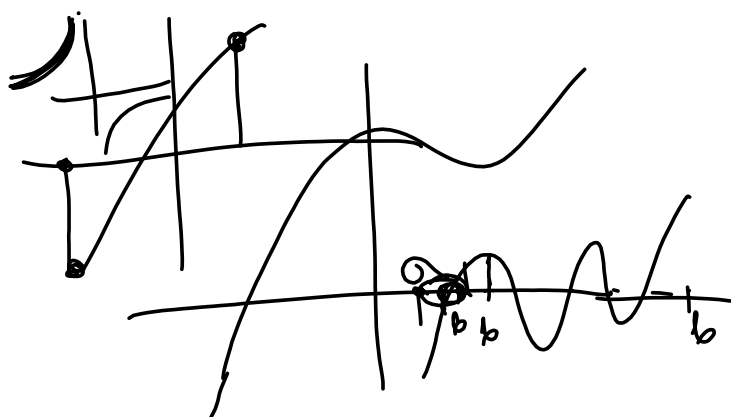
$$f) f(x) = \begin{cases} \frac{\sqrt{12+4x^2}-4}{x-1} & , \text{ je\u017bli } x \neq 1 \\ 1 & \text{ je\u017bli } x=1 \end{cases} \quad x_0 = 1$$

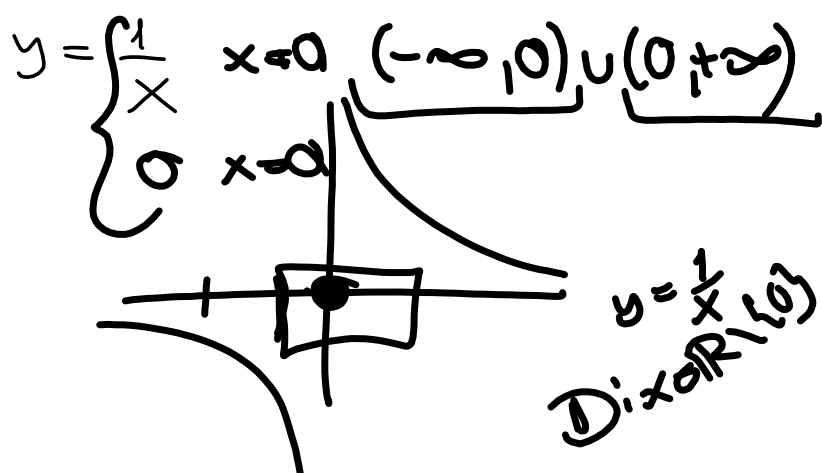
$$\begin{aligned} \lim_{x \rightarrow 1} \frac{\sqrt{12+4x^2}-4}{x-1} &= \lim_{x \rightarrow 1} \frac{\sqrt{12+4x^2}-4}{x-1} \cdot \frac{\sqrt{12+4x^2}+4}{\sqrt{12+4x^2}+4} = \lim_{x \rightarrow 1} \frac{12+4x^2-16}{(x-1)(\sqrt{12+4x^2}+4)} = \\ &= \lim_{x \rightarrow 1} \frac{4(x-1)(x+1)}{(x-1)(\sqrt{12+4x^2}+4)} = \frac{8}{8} = 1 \end{aligned}$$

$$\lim_{x \rightarrow 1} f(x) = f(1)$$

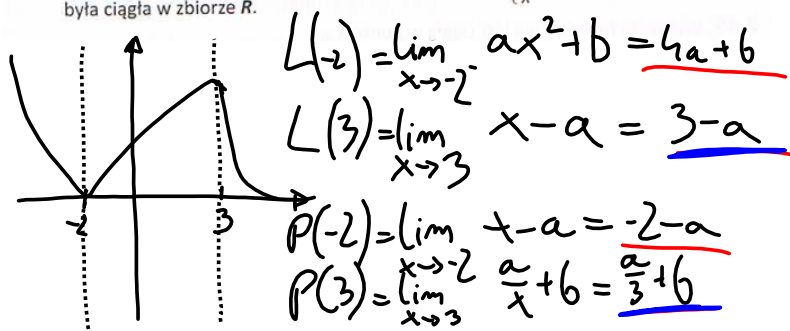
DARBOUX

 $f: \langle a, b \rangle \rightarrow \mathbb{R}$ *cyfa v dnuh.* $f(a) \cdot f(b) < 0 \Rightarrow \exists c \in (a, b) :$ $f(c) = 0$ 





2.50. Wyznacz parametry a, b tak, aby funkcja $f(x) = \begin{cases} ax^2 + b, & \text{jeśli } x \in (-\infty, -2) \\ x - a, & \text{jeśli } x \in [-2, 3) \\ \frac{a}{x} + b, & \text{jeśli } x \in (3, +\infty) \end{cases}$ była ciągła w zbiorze $\mathbb{R}$.



$$\begin{cases} 3-a = \frac{a}{3} + b \\ 4a+b = -2-a \end{cases}$$

$$b = -2 - 5a = -2 + 5\left(\frac{15}{11}\right) = -2 - \frac{75}{11}$$

$$3-a = \frac{a}{3} - 2 - 5a \quad | \cdot 3$$

$$9-3a = a-6-15a$$

$$11a = -15$$

$$a = -\frac{15}{11}$$

$$f(-2) = -2 + \frac{15}{11} \leftarrow \lim$$

$$f(3) =$$

$$\lim_{x \rightarrow -2} f(x) = f(-2) \Rightarrow f \text{ jest ciągła w } -2$$

$$\lim_{x \rightarrow 3} f(x) = f(3) \Rightarrow \text{---||---} 3$$

$$\text{dla } a = -\frac{15}{11} \quad b = -2 - \frac{75}{11}$$

2.51. Wyznacz a, b tak, aby funkcja $f(x) = \begin{cases} a, & \text{jeśli } x = -2 \\ \frac{4-x^2}{3-\sqrt{x+7}}, & \text{jeśli } x \in (-2, 2) \\ b, & \text{jeśli } x = 2 \end{cases}$ była ciągła w przedziale $\langle -2, 2 \rangle$.

$$f(x) = \begin{cases} a, & \text{jeśli } x = -2 \\ \frac{4-x^2}{3-\sqrt{x+7}}, & \text{jeśli } x \in (-2, 2) \\ b, & \text{jeśli } x = 2 \end{cases}$$

$\lim_{x \rightarrow -2^+} f(x) = a$

$$= \lim_{x \rightarrow -2^+} \frac{4-x^2}{3-\sqrt{x+7}} = \frac{\lim_{x \rightarrow -2^+} (2-x)(2+x)}{\lim_{x \rightarrow -2^+} (3-\sqrt{x+7})} =$$

$$= \lim_{x \rightarrow -2^+} (2+x)(3+\sqrt{x+7}) =$$

$$= 0(3+\sqrt{5}) = a$$

$$L = \lim_{(2) \ x \rightarrow 2} \frac{4-x^2}{3-\sqrt{x+7}} = 24 \Rightarrow b=24$$