$Z: a, b, c \in \mathbb{R}$

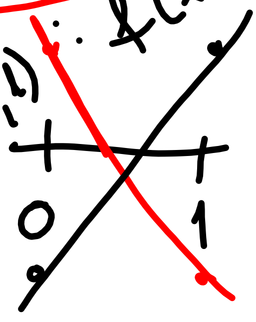
$$\underbrace{(a+b+c)}_{f(1)} \cdot \underbrace{c}_{f(0)} < 0 \qquad a \neq 0$$

$f(1)$

$T: \Delta > 0 \qquad b^2 > 4ac$ $\quad \Delta$

$f(x) = ax^2 + bx + c$

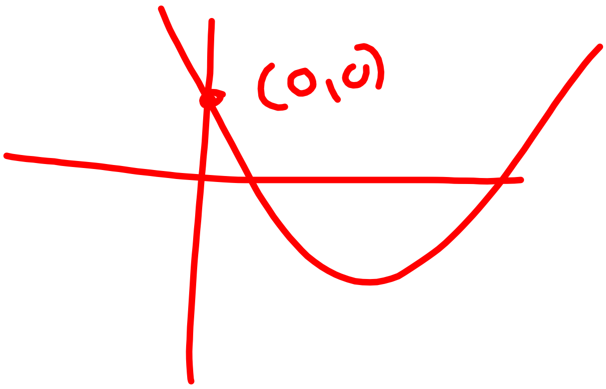
$f(1) \cdot f(0) = 0 \Rightarrow \exists x_1 \in (0,1): f(x_1) = 0 \wedge \exists x_2 \neq x_1: f(x_2) = 0 \quad x_2 \notin (0,1) \Rightarrow \Delta > 0$



lis 6-07:33

$f(x) = ax^2 + bx + c$

$f(0) = c \qquad (0, c)$



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$$T(\underbrace{T(x)})$$

$$T(x) = x^2 + 4x + 2 = \underbrace{(x+2)^2 - 2}$$

lis 6-07:53

Zadanie 4

Niech $T(x) = x^2 + 4x + 2$. Znaleźć wszystkie rozwiązania równania $T(T(T(x))) = 0$.

$$T(x) = (x+2)^2 - 2$$

$$T(T(x)) = \left((x+2)^2 - 2 + 2\right)^2 - 2$$

$$T(T(x)) = (x+2)^4 - 2$$

$$T(T(T(x))) = \left((x+2)^4 - 2 + 2\right)^2 - 2$$

$$T(T(T(x))) = \underline{(x+2)}^8 - 2 = 0$$

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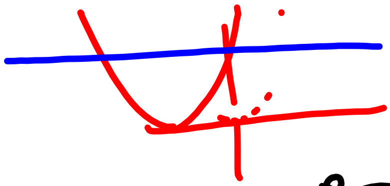
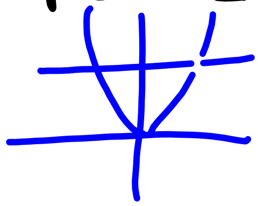
$$(x+2)^8=2$$



$$x+2=\sqrt[8]{2} \quad \vee \quad x+2=-\sqrt[8]{2}$$

$$x=\sqrt[8]{2}-2 \quad \vee \quad x=-\sqrt[8]{2}-2$$

$$t^8=2$$



lis 6-08:00

Zadanie 5

Wyznaczyć wszystkie pary (b, c) liczb rzeczywistych, dla których trójmian kwadratowy x^2+bx+c ma dwa różne pierwiastki i są nimi b i c.

$$\begin{cases} \Delta_x > 0 \\ b+c=-b \\ b\cdot c=c \end{cases}$$

$$c=-2b$$

$$bc-c=0$$

$$c(b-1)=0$$

$$\begin{cases} c=0 \\ b=0 \\ b\neq c \end{cases} \quad \vee \quad \begin{cases} b=1 \\ c=-2 \end{cases}$$

$$\emptyset$$

lis 6-08:04

Zadanie 6

Wykorzystując funkcję kwadratową $f(x) = (a_1x + b_1)^2 + (a_2x + b_2)^2 + \ldots + (a_nx + b_n)^2$, udowodnić nierówność Cauchy'ego-Schwarza $(a_1^2 + a_2^2 + \ldots + a_n^2)(b_1^2 + b_2^2 + \ldots + b_n^2) \geqslant (a_1b_1 + a_2b_2 + \ldots + a_nb_n)^2$.

lis 6-08:24

$f(x) = (a_1x+b_1)^2 + (a_2x+b_2)^2$

$(a_3x+b_3)^2 + \ldots + (a_mx+b_m)^2 \geq 0$

$\Delta_x \leq 0$ $\forall x \in \mathbb{R}$

$f(x) = x^2 \sum_{k=1}^{m} a_k^2 + x\,2\sum_{k=1}^{m} a_k b_k + \sum_{k=1}^{m} b_k^2$

$\Delta_x = 4\left(\sum a_k b_k\right)^2 - 4 \cdot \sum a_k^2 \cdot \sum b_k^2$

lis 6-08:13

Z: $m, n, \frac{m^2}{n} + \frac{n^2}{m} \in \mathbb{Z}$

T: $\frac{m^2}{n} \in \mathbb{Z}$

$p \mid m \cdot n$

$q \mid 1$

$\frac{p}{q} = p$

D: $\frac{m^2}{n} \cdot \frac{n^2}{m} = m \cdot n \in \mathbb{Z}$

$$f(x) = x^2 - \left(\frac{m^2}{n} + \frac{n^2}{m}\right)x + mn$$

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$$W(x) = a_n x^n + \dots + a_0$$

$\frac{p}{q} \in p$

$q \mid 1 \Rightarrow q \in \{\pm 1\}$

$a_i \in \mathbb{Z}$

$p \mid a_0$

$q \mid a_n$

$x_i \in \left\{ \frac{p}{q} : p \mid a_0 \wedge q \mid a_n \right\}$

$x^2 - 10x + 24 = 0$

$p \mid 24 \Rightarrow p \in \{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24\}$

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112 10,5

1, 112
2, 56
4, 28
7, 16
8, 14

lis 6-08:38

$$f(x)=\left(x-\frac{m^2}{n}\right)\left(x-\frac{n^2}{m}\right)$$

lis 6-08:32

lis 6-08:23