

⑦ $x = \frac{81^{-\frac{3}{4}} \cdot \sqrt{12}}{\sqrt[3]{72}}$

$$= \frac{(3^4)^{-\frac{3}{4}} \cdot 2 \cdot 3^{\frac{1}{2}}}{2 \cdot 3^{\frac{1}{3}}} = 3^{-3 + \frac{1}{4} - \frac{1}{3}} = 3^{-3 - \frac{1}{6}} = 3^{-\frac{19}{6}}$$

1.4. Która z liczb m i n jest większa, jeśli:

a) $m = 5^{12} + 2^{12}$, $n = 5^{12} - 2^{12}$

b) $m = 4^{12} - 9^6$, $n = \frac{2^{12} + 3^{12}}{2}$

$4^{12} - 9^6 = (2^{12})^2 - (3^6)^2 = (2^{12} - 3^6)(2^{12} + 3^6) =$

$a^2 - b^2 = (a-b)(a+b)$

$\underbrace{2^{12} - 3^6}_{>1} > 1$

$(4^6)^2 - (3^6)^2 = \underline{16^3 - 9^3}$

2.2. Übung

a) $(\lambda, \delta)^T = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$, $\begin{pmatrix} x^2 & x^2 & x^2 & x^2 \\ x^2 & x^2 & x^2 & x^2 \end{pmatrix}$ bi) $\begin{pmatrix} 0^2 - 0^2 \\ 0^2 + 0^2 \end{pmatrix}$
 $\left(\frac{0^2 - 0^2}{(0^2 - 0^2)^2 + (0^2 + 0^2)^2} \right)$ $\left(\frac{0^2 - 0^2}{(0^2 - 0^2)^2 + (0^2 + 0^2)^2} \right)$

c) $\begin{pmatrix} 6 \cdot 13^2 \\ \sqrt{6 \cdot 13} \cdot \sqrt{6 \cdot 13} \end{pmatrix}$ d) $\left((1 - \sqrt{2})^2 + (2 + \sqrt{2})^2 \right)^{\frac{1}{2}} \cdot \left(\frac{12^2}{16} - 28^2 \right)$

1.8. Parabeln (10 Punkte)

a) $x = 1024t^2$, $y = 8t^3$ bi) $x = \left(\frac{1}{27} \right)^{\frac{1}{3}}$, $y = 3420^{\frac{1}{3}}$

c) $x = 12 \cdot \sqrt[3]{27}$, $y = \left(\frac{28}{24} \right)^{\frac{1}{3}}$ d) $x = \left(\frac{361}{3027} \right)^{\frac{1}{3}}$, $y = (2793)^{\frac{1}{3}}$

1.9. Operatoren (10 Punkte)

a) $x = 85,128t^2$, $y = 64t^3$, $t = 128$, $t = (85,128)^{\frac{1}{2}}$, $y = \sqrt[3]{64}$

bi) $x = \left(\frac{1}{10} \right)^{\frac{1}{3}}$, $y = \left(\frac{1}{10} \right)^{\frac{1}{3}}$, $x = 85t^2$, $t = 2487$, $y = \left(\frac{1}{2} \right)^{\frac{1}{3}}$

1.10. ODEs (10 Punkte)

a) $y' = (\sqrt{2} + \sqrt{2}) \sqrt{y - \sqrt{2}} - \sqrt{2}$

bi) $e^{2t} \ln x = 2t^2 + \frac{x}{e^{2t} \ln(2\sqrt{2})}$

c) $\frac{\sqrt{2} \pm 1}{(\sqrt{2} - \sqrt{2} + 1)^2} \cdot \left(\frac{1}{2} \right)^{\frac{1}{2}} \cdot \left(\frac{1}{\sqrt{2}} \right)$