e) $\underbrace{1+\sin x+\sin^2 x+\sin^3 x+\ldots}_{} = \dfrac{4}{1+\mathrm{tg}^2 x}\left(1-\sin x+\sin^2 x-\sin^3 x+\ldots\right);$

$$\begin{cases} S=\dfrac{a_1}{1-q} \\ |q|<1 \end{cases}$$

f) $1+\cos x+\cos^2 x+\cos^3 x+\ldots = \dfrac{4}{1+\mathrm{ctg}^2 x}\left(1-\cos x+\cos^2 x-\cos^2 x+\ldots\right);$

g) $\cos x+\cos^3 x+\cos^5 x+\ldots = \dfrac{1}{\sin x};$

$\underline{1}$

h) $\sin x+\sin^3 x+\sin^5 x+\ldots = \dfrac{1}{\cos x};$

i) $\sin x+\sin^3 x+\sin^5 x+\ldots = \mathrm{tg}^2 x;$

j) $\cos x+\cos^3 x+\cos^5 x+\ldots = \mathrm{ctg}^2 x.$

$$S=\lim_{n\to\infty} S_n = \lim \frac{1-q^n}{1-q}\cdot a_1$$

☆ **4.14.** Rozwiąż równania:

a) $\dfrac{1-\sin x+\ldots+(-1)^n\sin^n x+\ldots}{1+\sin x+\ldots+\sin^n x+\ldots} = \dfrac{1-\cos 2x}{1+\cos 2x};$

b) $\dfrac{1-\cos 2x+\ldots+(-1)^n\cos^n 2x+\ldots}{1+\cos 2x+\ldots+\cos^n 2x+\ldots} = \dfrac{1}{3}\mathrm{tg}^4 x;$

c) $\dfrac{1-\mathrm{tg}\,x+\ldots+(-1)^n\mathrm{tg}^n x+\ldots}{1+\mathrm{tg}\,x+\ldots+(-1)^n\sin^n x+\ldots} = 1+\sin 2x;$

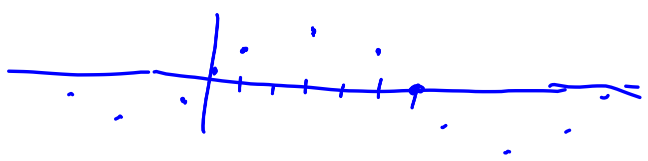
d) $\dfrac{1+\sin x+\ldots+\sin^n x+\ldots}{1-\sin x+\ldots+(-1)^n\sin^n x+\ldots} = \dfrac{4}{1+\mathrm{tg}^2 x};$

e) $\mathrm{tg}^2 x+\mathrm{tg}\,x = \sin 2x\left(1+\sin^2 x+\sin^4 x+\ldots\right).$

a) $\dfrac{1-\sin x+\ldots+(-1)^n\sin^n x+\ldots}{1+\sin x+\ldots+\sin^n x+\ldots}=\dfrac{1-\cos 2x}{1+\cos 2x};$

$\cos 2x = \cos^2 x - \sin^2 x$

c) $\dfrac{1-\mathrm{tg}\,x+\ldots+(-1)^n\mathrm{tg}^n x+\ldots}{1+\mathrm{tg}\,x+\ldots+(-1)^n\sin^n x+\ldots}=1+\sin 2x;$



d) $\lim_{x\to+\infty}\left(\frac{2x^2-x}{x+1}-ax-b\right)=0?$ c) $\lim_{x\to+\infty}\left(\sqrt{x^2+x+1}+ax-b\right)=0;$

$\infty \mp \infty$

$a, b = ?$

$$\frac{2x^2-ax^2-x-ax-bx-b}{x+1}=0$$

$x^2(2-a)-x(1+a+b)$

$L: \frac{\to 3}{x+1}$

$$\frac{x^2(2-a)-x(a+1+b)-b}{x+1}$$

$\boxed{a=2}$

$b=-3$

c) $\lim_{x\to+\infty}\left(\sqrt{x^2+x+1}+ax-b\right)=0;$ L: $\boxed{a<0}$

$x^2+x+1-(ax-b)^2$

$x^2+x+1-a^2x^2+2abx-b^2$

$\sqrt{x^2+x+1}-x-\frac{1}{2}$

$$\left(\frac{1}{\underbrace{\sqrt{x^2+x+1}}_{\infty}+\underbrace{x+\frac{1}{2}}_{\infty}}\right)=\left(\frac{1}{\infty}\right)$$

*h) $\lim\limits_{x \to +\infty} \left(\sin\sqrt{x+1} - \sin\sqrt{x} \right).$

$\sin\alpha \overset{-}{+} \sin\beta =$

$2 \sin\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2}$

b) $\lim\limits_{x \to 0} \frac{x - \sin 2x}{x - \sin 5x};$

$n \to \infty$ $\frac{\sin\frac{1}{n}}{\frac{1}{n}} = 1$

k) $\lim\limits_{x \to +\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}};$

b) $\lim\limits_{x \to +\infty} \dfrac{\sqrt{x^2+2} - \sqrt[3]{x^3+1}}{\sqrt[4]{x^4+8}}$;

d) $\lim\limits_{x \to +\infty} \dfrac{\sqrt{x^2+4} + \sqrt[4]{16x^4+1}}{\sqrt[5]{x^5+2}}$;

7.6. Oblicz:

a) $\lim\limits_{n \to \infty} \left(\dfrac{5}{6} + \dfrac{13}{36} + \ldots + \dfrac{2^n+3^n}{6^n} \right)$;

b) $\lim\limits_{n \to \infty} \left(\dfrac{7}{10} + \dfrac{29}{10^2} + \ldots + \dfrac{5^n+2^n}{10^n} \right)$;

c) $\lim\limits_{n \to \infty} \left(\dfrac{7}{12} + \dfrac{25}{12^2} + \ldots + \dfrac{3^n+4^n}{12^n} \right)$.

b) $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x^2+2}-\sqrt[3]{x^3+1}}{\sqrt[4]{x^4+8}}$;

d) $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x^2+4}+\sqrt[4]{16x^4+1}}{\sqrt[5]{x^5+2}}$;

c) $\lim\left(\sqrt{x^2+x+1}+ax-b\right)=0$

$\lim\limits_{n\to\infty} \dfrac{x^2+x+1-a^2x^2-b^2+2axb}{\sqrt{x^2+x+1}-ax+b} = \lim\limits_{n\to\infty} = 0$

1

$\frac{1}{2}$

$x, 1, 0$