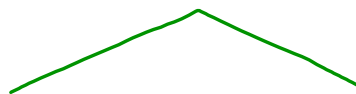


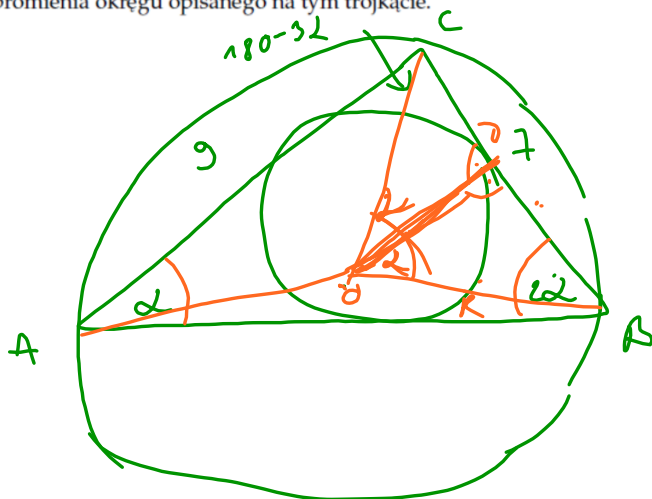
ZADANIE 8

W czworokącie $ABCD$ o obwodzie 24 dane są $|\angle ABC| = 120^\circ$ oraz $|BD| = 4\sqrt{3}$. Wiedząc, że środek przekątnej BD jest środkiem symetrii tego czworokąta oblicz jego pole.



ZADANIE 8

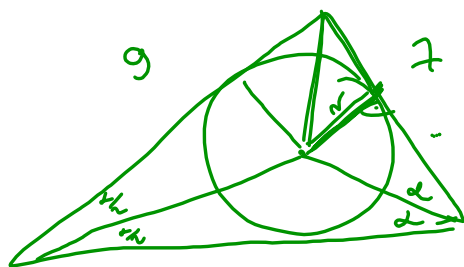
W trójkącie ABC dane są długości boków: $|AC| = 9$, $|BC| = 7$. Wiadomo też, że miara kąta $\angle ABC$ jest dwa razy większa od miary kąta $\angle BAC$. Oblicz stosunek długości promienia okręgu wpisanego w ten trójkąt do długości promienia okręgu opisanego na tym trójkącie.



$$\frac{r}{R} = ?$$

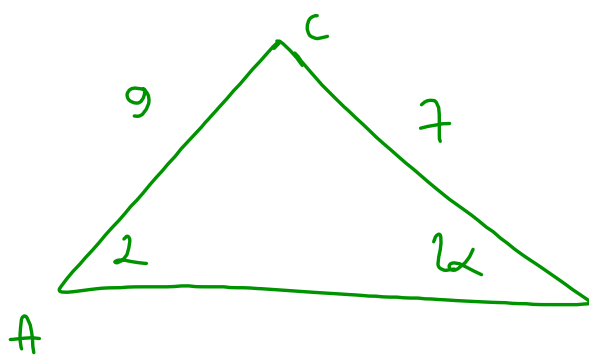
$$\angle BOI \Rightarrow \sin \angle = \frac{r}{R}$$

$$R = \frac{r}{\sin \angle}$$



$$P = p \cdot r$$

$$r = \frac{P_{\text{osc}}}{\frac{1}{2}(a+b+c)}$$



$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

TAB str 16

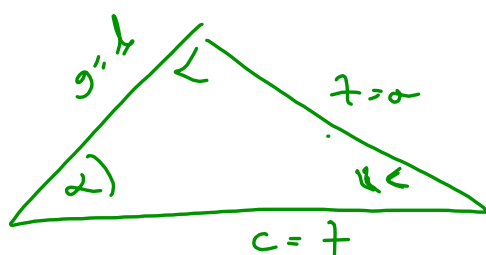
2 TU sin ABC

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

$$\frac{7}{\sin 2} = \frac{9}{\sin \alpha}$$

$$\frac{7}{\sin 2} = \frac{9}{2 \sin \alpha \cos \alpha}$$

$$\begin{aligned}
 14 \sin \angle \cos \angle &= 9 \sin \angle \\
 14 \sin \angle \cos \angle - 9 \sin \angle &= 0 \\
 14 \sin \angle \left(\cos \angle - \frac{9}{14} \right) &= 0 \\
 \underbrace{\sin \angle = 0} &\vee \underbrace{\cos \angle = \frac{9}{14}} \\
 \underline{\angle > 0^\circ} \vee \underline{\angle = 180^\circ}
 \end{aligned}$$



$$\cos \angle = \frac{9}{14}$$

$$a^2 = b^2 + c^2 - 2bc \cos \angle$$

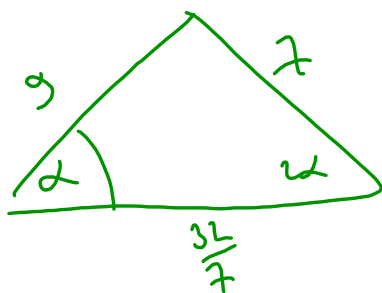
$$49 = 81 + c^2 - 14c \cdot \frac{9}{14}$$

$$c^2 - \frac{11}{7}c + 32 = 0 \quad / \cdot 7$$

$$\Delta = 81^2 - 4 \cdot 7 \cdot 224 = 189 = 17^2$$

$$7c^2 - 11c + 224 = 0$$

$$c = \frac{81 - 17}{14} = \frac{64}{14} = \frac{32}{7} = 4\frac{4}{7} \quad c_2 = \frac{98}{14} = 7$$



$$r = \frac{P}{\frac{1}{2}(16 + \frac{32}{7})} = \frac{P}{8 + \frac{16}{7}} = \frac{P}{10\frac{2}{7}}$$

$$\cos \alpha = \frac{9}{14}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin^2 \alpha = 1 - \frac{9^2}{14^2} = \frac{14^2 - 9^2}{14^2} = \frac{5 \cdot 13}{112}$$

$$\left(\sin \alpha = \frac{\sqrt{115}}{14} \right)$$

$$P = \frac{1}{2} \cdot 9 \cdot \frac{32}{7} \cdot \frac{\sqrt{115}}{14} =$$

$$r = \frac{\frac{1}{2} \cdot 9 \cdot \frac{32}{7} \cdot \frac{\sqrt{115}}{14}}{\frac{32 \cdot 8}{7}} = \frac{9\sqrt{115}}{16 \cdot 14}$$

$$\begin{array}{r|l} 115 & 5 \\ 23 & \end{array}$$

$$R = \frac{3,5}{\sin \alpha} = \frac{3,5 \cdot 14}{\sqrt{115}}$$

$$\frac{r}{R} = \frac{9\sqrt{115}}{16 \cdot 14} \cdot \frac{\sqrt{115}}{3,5 \cdot 14} = \frac{9 \cdot 115}{16 \cdot 14 \cdot 49}$$

$$SP \parallel \quad P = \frac{1}{2} ab \sin \gamma$$

$$P = \frac{1}{2} \underline{b} \cdot c \cdot \sin 2$$

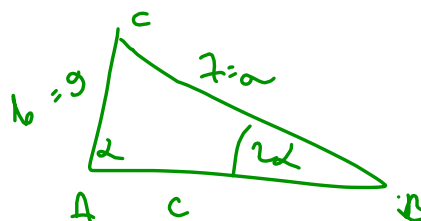
$$P = \frac{1}{7} c \cdot 2R \sin 2 \cos 2$$

$$P = 2R \cdot c \cdot \underbrace{\sin 2 \cos 2}_{\sin 4} = 1 - \cos 4$$

$$P = 2R \cdot \frac{32}{7} \cdot \frac{2}{7} \left(\frac{115}{175} \right) = \frac{72}{7} R \cdot \frac{125}{315}$$

||

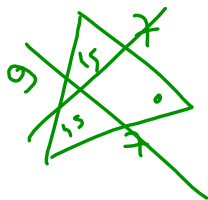
$$P = \checkmark \cdot \frac{12}{7}$$



$$\frac{b}{\sin 2} = 2R$$

$$\underline{b = 2R \sin 2}$$

$$\frac{2}{R} = \frac{115}{315}$$



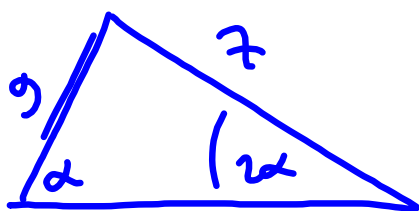
[TW]

np.

$$\sin 2\alpha = 2 \cdot \sin \alpha \cdot \cos \alpha \quad \left[\begin{array}{l} \text{TAB} \\ \text{sin i cos} \end{array} \right]$$

$$\sin 60^\circ = 2 \cdot \sin 30^\circ \cdot \cos 30^\circ$$

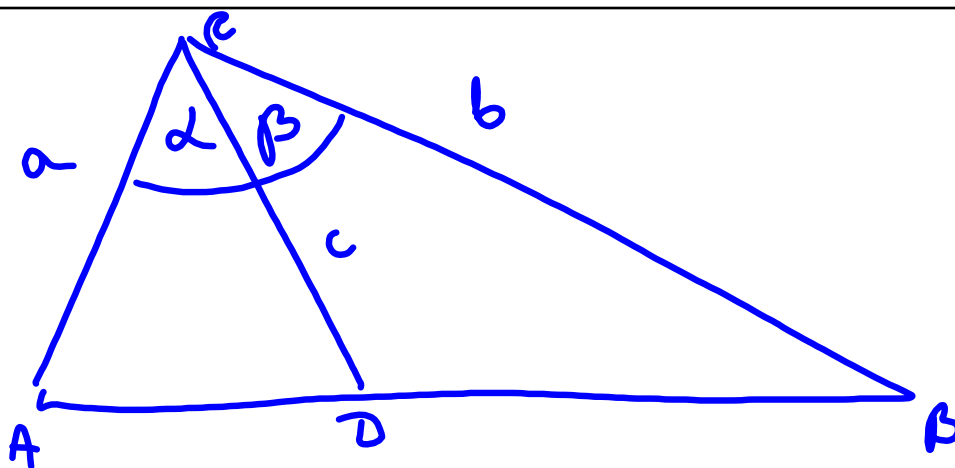
$$\left(\frac{\sqrt{3}}{2} \right) \quad \rightarrow \quad 2 \cdot \frac{1}{2} \cdot \left(\frac{\sqrt{3}}{2} \right)$$



2 TU SIN.

$$\frac{9}{\sin 2\alpha} = \frac{7}{\sin \alpha}$$

$$\frac{9}{2 \sin 2\alpha} = \frac{7}{\sin \alpha}$$



$$P_{ABC} = P_{ADC} + P_{BDC}$$

$$\frac{1}{2} a \cdot b \cdot \sin(\alpha + \beta) = \frac{1}{2} a c \sin \alpha + \frac{1}{2} c b \sin \beta$$

$$\sin(\alpha + \beta) = \left(\frac{c}{b}\right) \sin \alpha + \left(\frac{c}{a}\right) \sin \beta$$

$$\sin(\alpha + \beta) \Rightarrow \sin \alpha$$

$$\cos(\alpha + \beta) \Rightarrow \cos \alpha$$

$$\sin(\alpha - \beta)$$

$$\cos(\alpha - \beta)$$