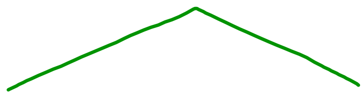
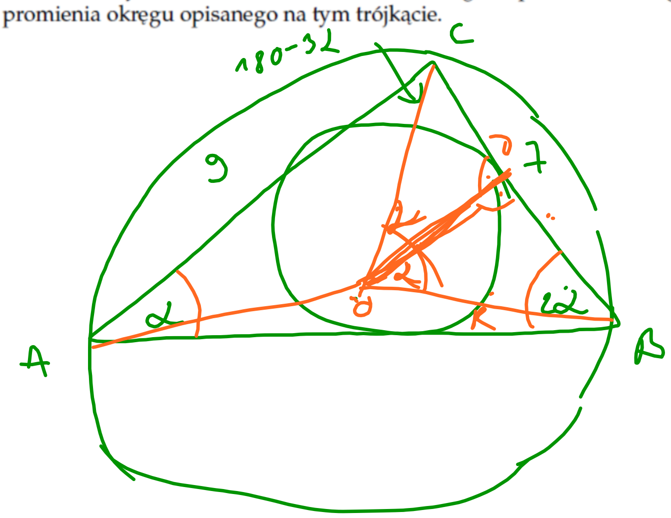
ZADANIE 8
W czworokącie $ABCD$ o obwodzie 24 dane są $|\angle ABC| = 120^\circ$ oraz $|BD| = 4\sqrt{3}$. Wiedząc, że środek przekątnej BD jest środkiem symetrii tego czworokąta oblicz jego pole.



ZADANIE 8
W trójkącie ABC dane są długości boków: $|AC| = 9$, $|BC| = 7$. Wiadomo też, że miara kąta $\angle ABC$ jest dwa razy większa od miary kąta $\angle BAC$. Oblicz stosunek długości promienia okręgu wpisanego w ten trójkąt do długości promienia okręgu opisanego na tym trójkącie.

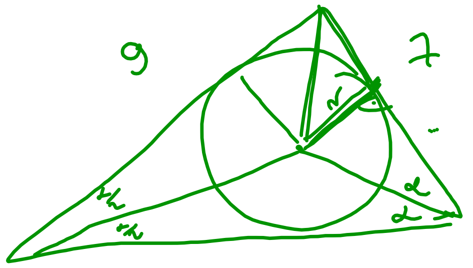


$\boxed{\frac{r}{R} = ?}$

$\angle \Delta BOD \Rightarrow$

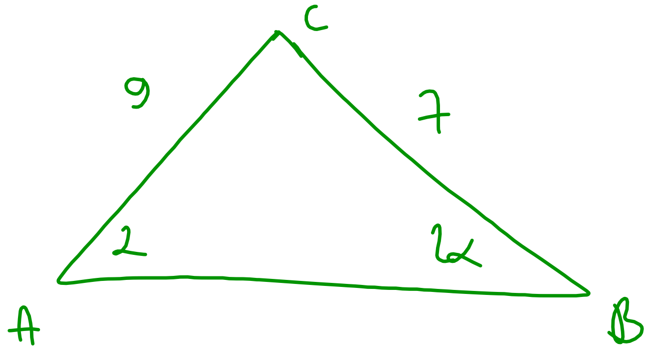
$\sin\alpha = \frac{3,5}{R}$

$\boxed{R = \frac{3,5}{\sin\alpha}}$



$$P = p \cdot r$$

$$r = \frac{P_{ABC}}{\frac{1}{2}(a+b+c)}$$



2 TU sin ABC

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\Updownarrow$$

$$\frac{7}{\sin\alpha} = \frac{9}{\sin 2\alpha}$$

$$\frac{7}{\sin\alpha} = \frac{9}{2\sin\alpha\cos\alpha}$$

$$\boxed{\sin 2\alpha = 2\sin\alpha\cos\alpha}$$

TAB STR 16

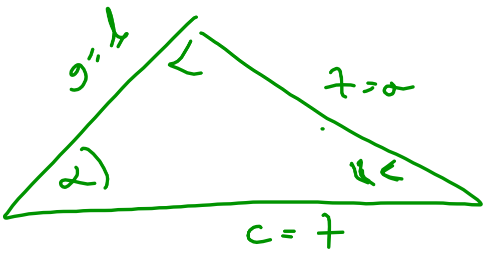
$$14\sin\alpha\cos\alpha = 9\sin\alpha$$

$$14\sin\alpha\cos\alpha - 9\sin\alpha = 0$$

$$14\sin\alpha\left(\cos\alpha - \frac{9}{14}\right) = 0$$

$$\cancel{\sin\alpha = 0} \lor \cos\alpha = \frac{9}{14}$$

$$\alpha = 0^\circ \lor \alpha = 180^\circ$$



$$\cos\alpha = \frac{9}{14}$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

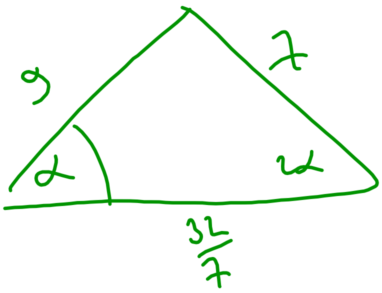
$$49 = 81 + c^2 - 18c \cdot \frac{9}{14}$$

$$c^2 - \frac{81}{7}c + 32 = 0 \quad / \cdot 7$$

$$7c^2 - 81c + 224 = 0$$

$$\Delta = 81^2 - 4 \cdot 7 \cdot 224 = 289 = 17^2$$

$$c = \frac{81-17}{14} = \frac{64}{14} = \frac{32}{7} = 4\frac{4}{7} \qquad \cancel{c_2 = \frac{98}{14} = 7}$$



$$r = \frac{P}{\frac{1}{2}\left(16+\frac{32}{7}\right)} = \frac{P}{8+\frac{16}{7}} = \frac{P}{10\frac{2}{7}}$$

$$\cos\alpha = \frac{9}{14}$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\sin^2\alpha = 1 - \frac{9^2}{14^2} = \frac{14^2-9^2}{14^2} = \frac{5\cdot 23}{14^2}$$

$$\boxed{\sin\alpha = \frac{\sqrt{115}}{14}}$$

$$P = \frac{1}{2}\cdot 9\cdot\frac{32}{7}\cdot\frac{\sqrt{115}}{14} =$$

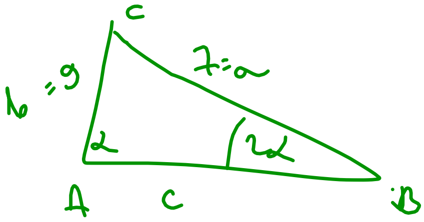
$$r = \frac{\frac{1}{2}\cdot\cancel{9}\cdot\frac{9}{\cancel{7}}\cdot\frac{\sqrt{115}}{14}}{\frac{\cancel{72}\,8}{\cancel{7}}} = \boxed{\frac{9\sqrt{115}}{16\cdot 14}}$$

$115 \mid 5$
$23 \mid$

$$R = \frac{3{,}5}{\sin\alpha} = \frac{3{,}5\cdot 14}{\sqrt{115}}$$

$$\frac{r}{R} = \frac{9\sqrt{115}}{16\cdot 14}\cdot\frac{\sqrt{115}}{3{,}5\cdot 14} = \boxed{\frac{9\cdot 115}{16\cdot 14\cdot 49}}$$

SP II $P=\frac{1}{2}ab\sin\gamma$



$P=\frac{1}{2}b\cdot c\cdot\sin\alpha$

$P=\frac{1}{\cancel{2}}c\cdot\cancel{2}R\sin 2\alpha\cos\alpha$

$P=2R\cdot c\cdot\cos\alpha\,\sin^2\alpha$ $\quad \sin^2\alpha = 1-\cos^2\alpha$

$P=2R\,\frac{32}{7}\,\frac{9}{\cancel{14}_{7}}\left(\frac{115}{196}\right)=\frac{72}{7}R\cdot\frac{115}{343}$

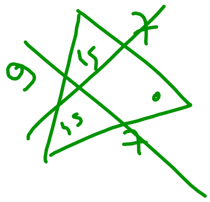
$\frac{b}{\sin 2\alpha}=2R$

$b=2R\sin 2\alpha$

$\parallel$

$P=r\cdot\frac{72}{7}$

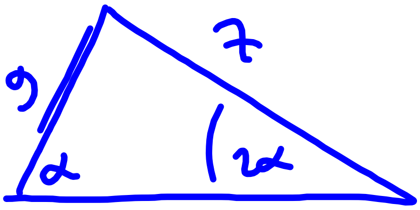
$\frac{r}{R}=\frac{115}{343}$



$\boxed{TW}$ $\boxed{\sin 2\alpha = 2\cdot\sin\alpha\cdot\cos\alpha}$ $\boxed{\text{TAB str 1 i 5}}$

np. $\sin 60^\circ = 2\cdot\sin 30^\circ\cdot\cos 30^\circ$

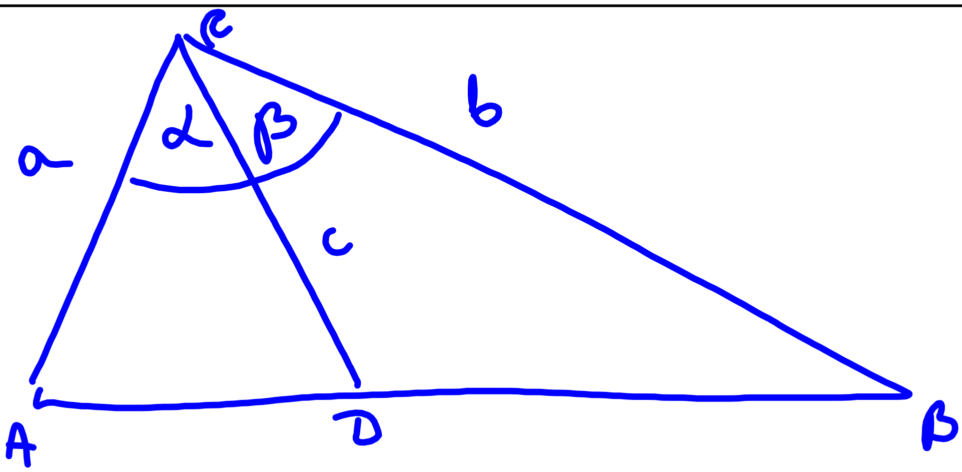
$\boxed{\frac{\sqrt{3}}{2}} = \cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\boxed{\frac{\sqrt{3}}{2}}$



2 TU SIN.

$$\frac{9}{\sin 2\alpha} = \frac{7}{\sin\alpha}$$

$$\frac{9}{2\sin\alpha\cos\alpha} = \frac{7}{\sin\alpha}$$



$$P_{ABC} = P_{BDC} + P_{ADC}$$

$$\|$$

$$\cancel{\tfrac{1}{2}}\, a\cdot b\cdot\sin(\alpha+\beta) = \cancel{\tfrac{1}{2}}\, ac\sin\alpha + \cancel{\tfrac{1}{2}}\, cb\sin\beta$$

$$\sin(\alpha+\beta) = \boxed{\frac{c}{b}}\cdot\sin\alpha + \boxed{\frac{c}{a}}\sin\beta$$

$\sin(\alpha+\beta) \Rightarrow \sin 2\alpha$

$\cos(\alpha+\beta) \Rightarrow \cos 2\alpha$

$\sin(\alpha-\beta)$

$\cos(\alpha-\beta)$