ZADANIE 15 (7 PKT)

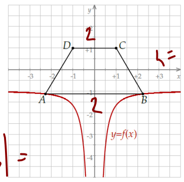
Rozpatrujemy wszystkie trapezy $ABCD$, których wierzchołki A i B leżą na wykresie funkcji $f(x) = -\frac{2}{3}\cdot\frac{1}{x^2} - 1$ określonej dla $x \neq 0$. Punkt C ma współrzędne $(1,1)$, a oś Oy jest osią symetrii tego trapezu (zobacz rysunek).

$\boxed{D: x > 0}$

Oblicz obwód tego trapezu $ABCD$, którego pole jest najmniejsze możliwe.

$C(1,1)$ $D(-1,1)$ $B\left(x, -\frac{2}{3}\cdot\frac{1}{x^2}-1\right)$

$A\left(-x, -\frac{2}{3}\cdot\frac{1}{x^2}-1\right)$



$|\vec{DC}| = 2$ $|\vec{AB}| = 2x$ $h = |y_c - y_B| =$

$\left|\frac{2}{3}\cdot\frac{1}{x^2}+2\right| = \frac{2}{3}\frac{1}{x^2}+2$

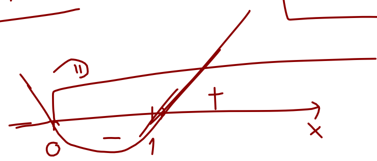
$\begin{cases} P(x) = (1+x)\cdot\left(\frac{2}{3}\cdot\frac{1}{x^2}+2\right) = 2x+2+\frac{2}{3}\frac{1}{x}+\frac{2}{3}\cdot\frac{1}{x^2} \\ x \in \mathbb{R}_+ \end{cases}$

$P'(x) = 2 - \frac{2}{3}\cdot\frac{1}{x^2} - \frac{4}{3}\cdot\frac{1}{x^3} = \frac{6x^3-2x-4}{3x^3}$

$\boxed{(x^{-2})' \quad -2x^{-3} = -2\frac{1}{x^3}}$

$P'(x) = \frac{2(x-1)(3x^2+3x+2)}{3x^3}$

x	$(0,1)$	1	$(1,+\infty)$
P'	$-$	0	$+$
P	$\searrow$		$\nearrow$



y_{min} M $x = 1$

$B\left(1, -\frac{5}{3}\right)$

$A\left(-1, -\frac{5}{3}\right)$

$Obw = 9\frac{1}{3}$

6.67. $y=\frac{x^2-2x+3}{x^2+2x-3}$.

6.68. $y=\frac{3}{(1-x^2)(1-2x^3)}$.

6.69. $y=\frac{\sqrt[3]{x}}{1-\sqrt[3]{x}} = \frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}} = \frac{x}{x^{\frac{1}{3}}-x}$

6.70. $z=\frac{1+\sqrt{t}}{1+\sqrt{2t}}$.

6.71. $s=(3t+1)^7$.

6.72. $v=(4z^2-5z+13)^5$.

6.73. $x=\left(\frac{1}{t}+4\right)^4$.

6.74. $s=\left(7t^2-\frac{4}{t}+6\right)^6$.

69
70
71

$y' = \frac{\frac{1}{x^{\frac{2}{3}}} - \frac{1}{x^{\frac{1}{3}}}}{3\cdot(1-x^{\frac{1}{3}})^2} = \frac{x^{\frac{1}{3}} - x^{\frac{2}{3}}}{3(x^{\frac{1}{2}}-x^{\frac{2}{3}})^2}$

$y = \frac{(x^{\frac{1}{3}})'\cdot(1-x^{\frac{1}{3}}) - (1-x^{\frac{1}{3}})'\cdot x^{\frac{1}{3}}}{(1-x^{\frac{1}{3}})^2} =$

$\frac{\frac{1}{3}x^{-\frac{2}{3}}(1-\cancel{x^{\frac{1}{3}}}) + \frac{1}{3}\cancel{x^{-\frac{2}{3}}\cdot x^{\frac{1}{3}}}}{(1-x^{\frac{1}{3}})^2} = \frac{x^{-\frac{2}{3}}\cdot x^{+\frac{2}{3}}}{3(1-x^{\frac{1}{3}})^2\cdot x^{\frac{2}{3}}}$

$\frac{1}{3(x^{\frac{1}{3}}-x^{\frac{2}{3}})^2}$

$s=(3t+1)^7.$ $\quad s'=7(3t+1)^6\cdot(3t+1)'=7(3t+1)^6\cdot 3$

6.90. $s=\sqrt{\dfrac{1-\sqrt{t}}{1+\sqrt{t}}}.$

cos'

$s'(t)=\dfrac{1}{2\sqrt{\dfrac{1-\sqrt{t}}{1+\sqrt{t}}}}\cdot\left(\dfrac{1-\sqrt{t}}{1+\sqrt{t}}\right)'=$

$D:$ $t\geq 0$ $t<1$

$\dfrac{\sqrt{\cancel{1+\sqrt{t}}}}{2\sqrt{1-\sqrt{t}}}\cdot\dfrac{(1-\sqrt{t})'(1+\sqrt{t})-(1+\sqrt{t})'(1-\sqrt{t})}{\sqrt{\cancel{1+\sqrt{t}}}\sqrt{1+\sqrt{t}}+\sqrt{1+\sqrt{t}}^2}=$ $D: t\in\langle 0,1)$

$\dfrac{-\frac{1}{2\sqrt{t}}(1+\sqrt{t})-\frac{1}{2\sqrt{t}}(1-\sqrt{t})}{2(\sqrt{1-t}\cdot(1+\sqrt{t})}=$ $\quad -\dfrac{1}{2\sqrt{t}\sqrt{1-t}(1+\sqrt{t})}$

6.105. $y=\cos x-\frac{1}{3}\cos^3 x$.

6.107. $y=\mathrm{tg}^4\sqrt{x}$.

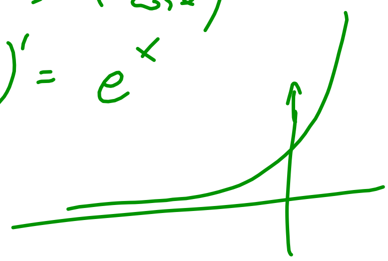
6.109. $y=e^{ax}(a\sin x-\cos x)$.

$(\sin x)'=\cos x$

$(\cos x)'=-\sin x$

$(\mathrm{tg}\,x)'=\left(\frac{\sin x}{\cos x}\right)'=\dots$

$(e^x)'=e^x$



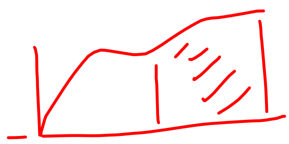
6.109. $y'=\left(e^{ax}(a\sin x-\cos x)\right)'=$

$(e^{ax})'\cdot(a\sin x-\cos x)+e^{ax}(a\sin x-\cos x)'=$

$e^{ax}\cdot a\cdot(a\sin x-\cos x)+e^{ax}(a\cos x+\sin x)=$

$=e^{ax}a^2\sin x-\cancel{ae^{ax}\cos x}+\cancel{e^{ax}a\cos x}+e^{ax}\sin x=$

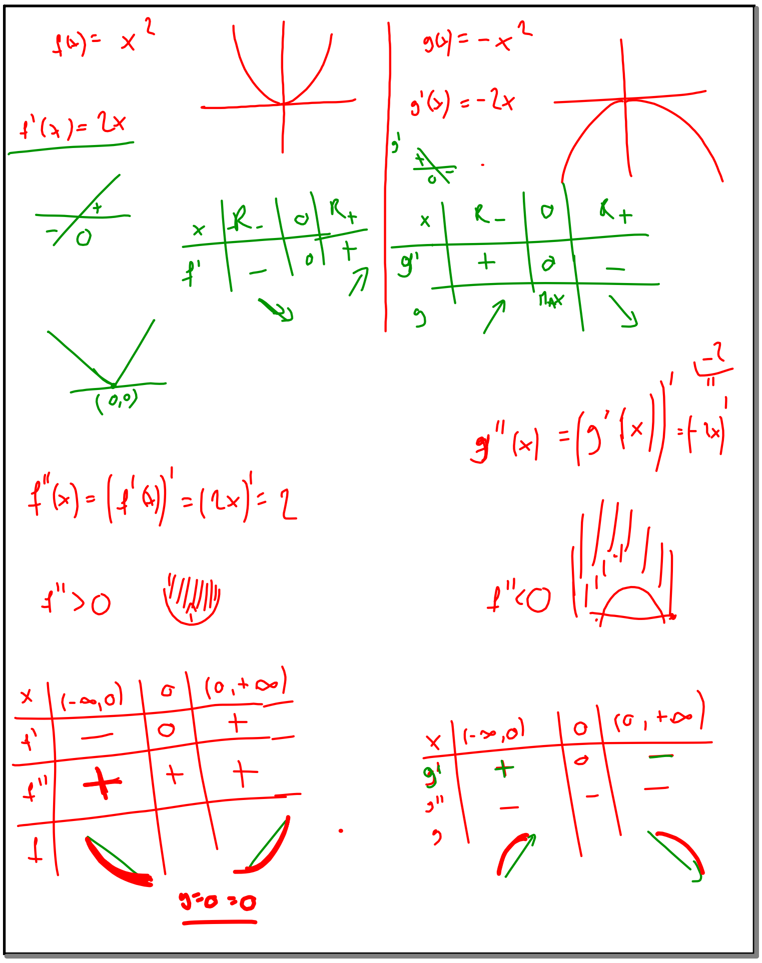
$=e^{ax}\sin x(a^2+1)$

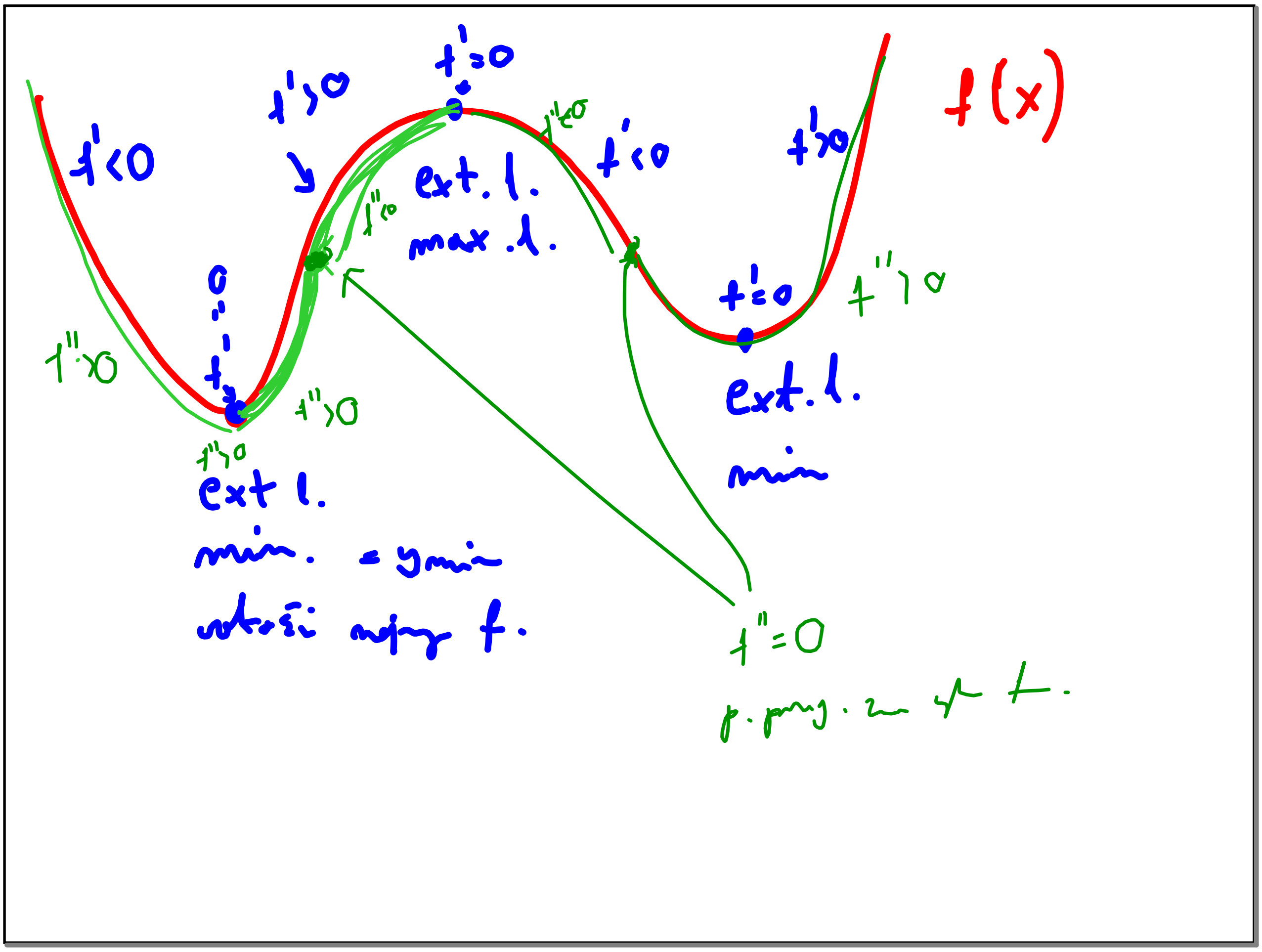


$\int e^{ax}\sin(a^2+1)\,dx=e^{ax}(a\sin x-\cos x)+C$

f'

$\int f(x)\,dx$


f(x) = x^2
f'(x) = 2x
g(x) = -x^2
g'(x) = -2x
f''(x) = (f'(x))' = (2x)' = 2
g''(x) = (g'(x))' = (-2x)' = -2
f''>0
f''<0


f(x)
f'<0
f'>0
f'=0
ext. l.
max. l.
f'<0
f'>0
f'=0
ext. l.
min
f''>0
ext l.
min.
f''=0

ZADANIE 10.19. Zbadać przebieg zmienności funkcji $y=\sin^2 x+\cos x$.

1) D: $x \in \mathbb{R}$

2) $\lim_{x\to\pm\infty}(\sin^2 x+\cos x)$ - nie istnieje

$\boxed{0 = \frac{0}{\infty}}$

3) asymptoty → pionowe nie ma

$y=ax+b$ $\quad \lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{\sin^2 x}{x}+\frac{\cos x}{x}=0$ $\quad \in(-1,1)$, $\to 0$

$a=0$

$b=\lim_{x\to\pm\infty}(f(x)-ax)=\lim_{x\to\pm\infty}f(x) \leftarrow$ nie ist.

4) OY: $x \Rightarrow 0 \Rightarrow f(x)=1$ $\quad$ OY: $(0,1)$

OX: $\sin^2 x+\cos x=0$ $\quad 1-\cos^2 x+\cos x=0$

5) P NP $\quad f(x)=\sin^2 x+\cos x$

(P) $f(-x)=\sin^2(-x)+\cos(-x)=\sin^2 x+\cos x=f(x)$

f'

f''

x	
x'	
x''	

10.93. $y=\frac{(x+1)^2}{2x}$. ✓

10.94. $y=\frac{x^2-x-4}{x-1}$.

10.95. $y=\frac{x^2+2x+25}{(x+1)^2}$.

10.96. $y=\frac{x^2+x+1}{x^2-1}$.

10.97. $y=\frac{15x^2-13x-20}{8x^2+10x-7}$.

10.98. $y=\frac{x^4}{2-x^3}$.

10.99. $y=\frac{x^3}{(x-1)^2}$.

10.100. $y=\frac{(x+3)^3}{(x+2)^2}$.

10.101. $y=x+2\sqrt{-x}$. ✓

10.102. $y=\sqrt[3]{x^2}-1$.

10.103. $y=1-\sqrt[3]{(x-4)^2}$.

10.104. $y=2x-3\sqrt[3]{x^2}$.

$\lim_{x\to x_0} f(x) = \pm\infty \Rightarrow$ as pion $x=x_0$

$(-\infty, x_0)\cup(x_0, x_1)(x_1,+\infty)$ $\boxed{\frac{1}{x}}$

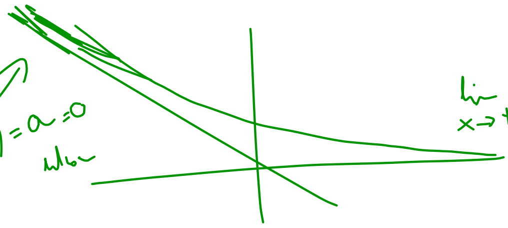
$\lim_{x\to\pm\infty} f(x) = b \Rightarrow$ as poziom $\boxed{y=b}$

$$\left.\begin{array}{l}\lim_{x\to\pm\infty}\frac{f(x)}{x}=a\in\mathbb{R}\\ \Downarrow \\ \lim_{x\to\pm\infty}(f(x)-ax)=b\end{array}\right\}\Rightarrow y=ax+b$$

as. ukośna

$\lim_{x\to-\infty} f(x)=a=0$

ukoś



$\lim_{x\to+\infty} f(x)=0$

poziom