Dziadek i babka mają razem 140 lat.
Dziadek ma dwa razy tyle ile babka wtedy,
kiedy on miał tyle, ile ona ma teraz.
Ile lat mają dziadek i babka?

d - wiek dziadka
b - wiek babki

$$\begin{cases} b+d=140 \\ d=2\cdot(b-(d-b)) \end{cases}$$

$d=140-b$

$d=2(b-d+b)$

$140-b=4b-2(140-b)$

$$\begin{cases} b=140-d \\ 3d=4b \end{cases}$$

$3d=4(140-d)$

$3d=560-4d$

$d=80$ $\quad b=60$

$Z: m>1 \quad m\in N_+ \quad \wedge x=-1 \wedge y=1$

$T: \quad \frac{x}{m+1}+\frac{y}{m-1}=\frac{2}{m^2-1}$ $\quad \boxed{m=2}$

$m=3$

$D: \quad \frac{-1}{2+1}+\frac{1}{2-1}=\frac{2}{2^2-1}$ $\quad L=P$

$\frac{-1}{1001}+\frac{1}{999}=\frac{2}{999999}$

$Z: m > 1 \quad m \in \mathbb{N}_+ \quad \wedge \underline{x = 1} \wedge \underline{y = 1}$

$T: \quad \frac{x}{m+1} + \frac{y}{m-1} = \frac{2}{m^2-1}$ $\quad \boxed{m=2}$ $\quad m=3$

$D: \quad \frac{-1}{m+1} + \frac{1}{m-1} \overset{?}{=} \frac{2}{m^2-1} \qquad \frac{1}{3} \quad \frac{1}{2}$

$L = \frac{-1}{m+1} + \frac{1}{m-1} = \frac{-(m-1)}{(m+1)(m-1)} + \frac{(m+1)}{(m+1)(m-1)} =$

$\frac{-m+1+m+1}{(m+1)(m-1)} = \frac{2}{m^2-1} = P$

39)

$2x + 4y = 18 \qquad x, y \in \mathbb{N}$

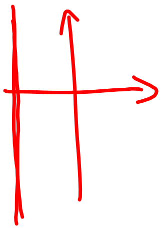
$x = 9 - 2y \qquad x, y \in \{0, 1, 2, 3, \ldots\}$

x y

(9, 0) (3, 3)

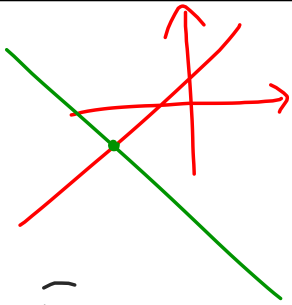
(7, 1) (1, 4)

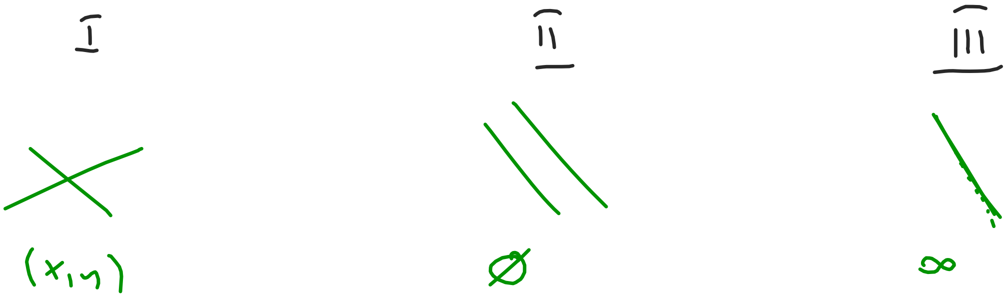
(5, 2) ~~5)~~

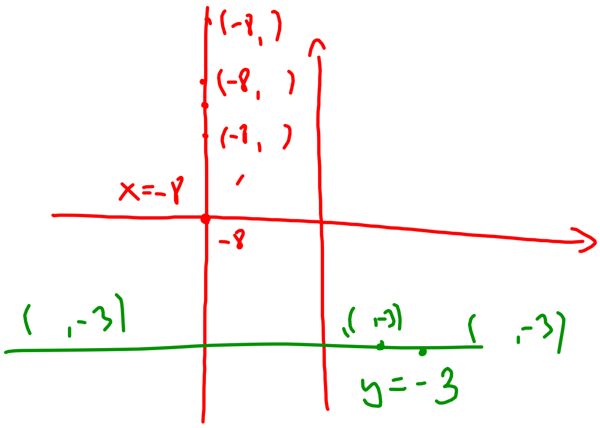


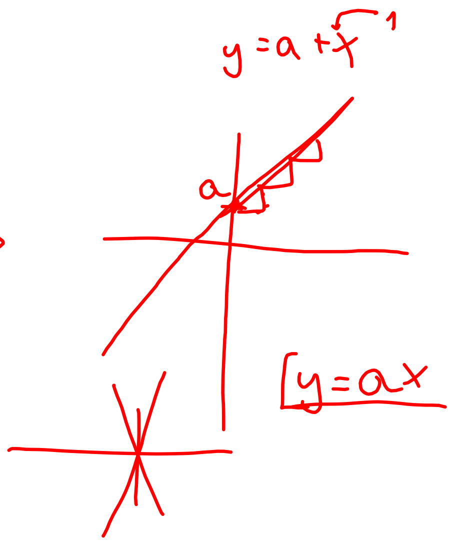
$$\begin{cases} a_1x+b_1y=c_1 \\ a_2x+b_2y=c_2 \end{cases}$$

równ. lin.









y=ax -proporcjonalność prosta (0,0)

y=x+b funkcja rosnąca po jednej kratce, przecina oś OY w punkcie (0,b)

$$\begin{cases} a_1x+b_1y=c_1 \\ a_2x+b_2y=c_2 \end{cases}$$

wsp wt.

Układ dwóch równań liniowych z dwoma niewiadomymi, w postaci uporządkowanej.

I

0 r.

$\Updownarrow$

ukł. sprz.

$\Updownarrow$

//

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\boxed{\ne\frac{c_1}{c_2}}$$

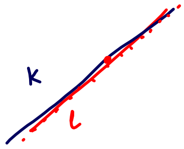
$$\begin{cases} 2x+3y=30 \\ 4x+6y=58 \end{cases}$$

∞

$\Updownarrow$

nieoznaczony

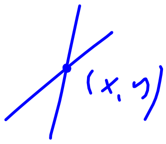
/



a_1

1 Rz.

oznaczony



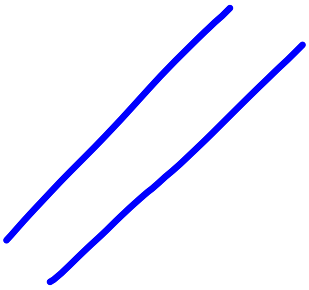
$$\begin{cases} 2x+3y=30 & \vert\cdot(-2) \\ 4x+6y=58 & \vert:(-2) \end{cases}$$

$$\begin{cases} -4x-6y=-60 \\ 4x+6y=58 \end{cases}$$

$$\boxed{\begin{cases} 2x+3y=30 \\ -2x-3y=-29 \end{cases}}$$

$\boxed{0=-2}$ sp

$\emptyset$



$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2}}$

$$\begin{array}{l} k \\ l \end{array}\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

I

wyznacznik główny

$$W = \begin{array}{cc} x & y \end{array}$$
$$W = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1 = 0$$

$\Downarrow$

proste opisane równaniami są $\parallel$

$a_1b_2 = b_1a_2$

$a_1b_2 - b_1a_2 = 0$ $\quad \cdot c$

$$W_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = c_1 \cdot b_2 - b_1 \cdot c_2 \neq 0$$

$\cdot c$

$$W_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = a_1c_2 - c_1a_2 \neq 0$$

Układ nieoznaczony

(x,y)

$$\begin{cases} 2x + 3y = 30 \\ 2x + 3y = 30 \end{cases} \qquad \begin{cases} 2x + 3y = 30 \\ 6x + 9y = 90 \end{cases}$$

$\Downarrow$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \qquad \begin{cases} y = -\frac{2}{3}x + 10 \\ x \in \mathbb{R} \end{cases}$$

$$\begin{cases} x = 6 \\ y = -\frac{2}{3} \cdot 6 = 6 \end{cases} \quad (6,6)$$

$$(3,8) \quad \begin{cases} x = 3 \quad (\text{np } 3) \\ y = -\frac{2}{3} \cdot 3 + 10 = 8 \end{cases}$$

$$\begin{cases} 2x + 3y = 30 \\ 6x + 9y = 90 \end{cases}$$

$$W = \begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} = 2\cdot 9 - 3\cdot 6 = 0$$

$$W_x = \begin{vmatrix} 30 & 3 \\ 90 & 9 \end{vmatrix} = 30\cdot 9 - 3\cdot 90 = 0$$

$$W_y = \begin{vmatrix} 2 & 30 \\ 6 & 90 \end{vmatrix} = 2\cdot 90 - 30\cdot 6 = 0$$

$$(W = 0 \wedge W_x = 0 \wedge W_y = 0)$$

$$\Downarrow$$

ukł. jest nieozn.

/ ∞

$$\begin{cases} a_1 x + b_1 y = c_1 \\ a_2 x + b_2 y = c_2 \end{cases}$$

wt.

$$W = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \underline{a_1 b_2 - b_1 a_2} \neq 0$$

1 rozw

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Updownarrow$$

$$a_1 b_2 \neq b_1 a_2$$

$$\underline{a_1 b_2 - b_1 a_2} \neq 0$$