## C. ARYTMETYCZNY

$(a_n)$ - ciąg arytm. $\iff$

$a_{n+1} = a_n + r$ $\quad \boxed{r \in \mathbb{R}}$

$\boxed{r = a_{n+1} - a_n}$

j) $a_n = \boxed{\frac{n^2-9}{n+3}} \Rightarrow a_{n+1} = \frac{(n+1)^2-9}{(n+1)+3} =$

$r = a_{n+1} - a_n = \frac{(n-3)(n+3)}{n+3}$

$a_n = n-3$

$a_{n+1} = n+1-3 = n-2$

$r = a_{n+1} - a_n = n-2-(n-3) = 1 \in \mathbb{R}$

$\Downarrow$

ciąg arytm.

bo r nie zależy od n

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$a_{n+1} = a_n + r$ $\quad (n \in \mathbb{N})$

$r = a_{n+1} - a_n$

$a_n = a_1 + (n-1)r$

$\boxed{a_n = a_k + (n-k)r}$

$a_n = a_0 + n \cdot r$

$\boxed{r = \frac{a_n - a_0}{n}}$

$r = 2$

$-19, -15, -13, \ldots, 121$

$a_1 \quad a_2 \quad a_n$

$n = \frac{121-(-19)}{2} + 1 = \frac{121+19}{2} + 1 = 70$

$-19, -17, \ldots, 111$

$111 + 19 + 1$

$111 - -19 = 130$

$a_1 \quad a_2$

$99, 100, 101, \ldots, 999$

$999 - 100 + 1$

$999 - 99 = 900$

$17, 19, \ldots, 51$

$51 - 17 + 1$

$a_1 = 17$

$a_n = 51$

$r = 1$

$a_n = a_1 + (n-1)r$

$\frac{(99-17)}{2} =$

$99, 100, 101, \ldots, 996$

$a_1 \quad a_2$

$\frac{996-100}{2} + 1$

$999 - 7 = 992$

$992 + 1 = 993$

$\boxed{996}$

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$k_0 -$

$k_1$

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$a_{m-1}, a_m, a_{m+1}$ - c. arytm.

$a_m - r, a_m, a_m + r$

$a_{m+1} - a_m = a_m - a_{m-1} = r$

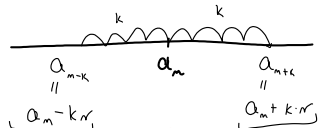
$2a_m = a_{m-1} + a_{m+1}$

Tj.

$\boxed{a_m = \frac{a_{m-1} + a_{m+1}}{2}}$ $\quad \boxed{m \in \mathbb{N}_+, m > 1}$

$\Downarrow$

$a_m$ jest arytm.




$\boxed{a_m = \frac{a_{m-k} + a_{m+k}}{2}}$

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$S_n = a_1 + a_2 + a_3 + \cdots + a_n =$

$= a_1 + (a_1 + r) + (a_1 + 2r) + \cdots + (a_1 + (n-1)r)$

$= n \cdot a_1 + (r + 2r + 3r + \cdots + (n-1)r) =$

$= n \cdot a_1 + r(1 + 2 + 3 + \cdots + (n-1))$

$-1 \quad -2 \quad +2 \quad 1$

$\frac{1}{2}n(n-1)$

$S_n = n \cdot a_1 + \frac{1}{2}n(n-1) \cdot r =$

$\boxed{S_n = \frac{2a_1 + (n-1)r}{2} \cdot n} =$

$\frac{a_1 + \overbrace{a_1 + (n-1)r}^{a_n}}{2} \cdot n = \boxed{\frac{a_1 + a_n}{2} \cdot n}$

$S_6 = a_1 + a_2 + a_3 + a_4 + a_5 =$ $\quad a_1 + a_5 = 5$

$\boxed{a_1 + a_5} + a_2 + a_4 + a_3 =$

$2a_3 + 2a_3 + a_3 = 5a_3 = 5\frac{a_1 + a_5}{2}$

$a_1 + a_2 + a_3 + a_4 + a_5 + a_6$

$\frac{a_1 + a_6}{2} \cdot 6 = S_6$

dla 1,10