

d) 22, 2,2, 0,22, ...

f) $-1\frac{3}{4}, \frac{1}{2}, -\frac{1}{7}, \dots$

Vorteil

$a_{n+1} = a_n \cdot q$

$a_0, a_1, a_2, \dots$

$\left\{ \begin{array}{l} a_1 = 0 \\ q = 0 \end{array} \right. \quad \text{g. r.}$

$\left\{ \begin{array}{l} a_1 = 5 \\ q = 1 \end{array} \right. \quad \text{d. r.}$

$q = 1 \rightarrow 10$

$5, 0, 0, 0, 0 \quad \left\{ \begin{array}{l} a_0 = 5 \\ q = 0 \end{array} \right.$

$a_2 = a_1 \cdot q$

$1 \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$

$a_0 = 1$
 $q = \frac{1}{2}$

$1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16} \quad \frac{1}{32} \quad \frac{1}{64} \quad \frac{1}{128} \quad \frac{1}{256} \quad \frac{1}{512} \quad \frac{1}{1024} \quad \frac{1}{2048} \quad \frac{1}{4096} \quad \frac{1}{8192} \quad \frac{1}{16384} \quad \frac{1}{32768} \quad \frac{1}{65536} \quad \frac{1}{131072} \quad \frac{1}{262144} \quad \frac{1}{524288} \quad \frac{1}{1048576} \quad \frac{1}{2097152} \quad \frac{1}{4194304} \quad \frac{1}{8388608} \quad \frac{1}{16777216} \quad \frac{1}{33554432} \quad \frac{1}{67108864} \quad \frac{1}{134217728} \quad \frac{1}{268435456} \quad \frac{1}{536870912} \quad \frac{1}{1073741824} \quad \frac{1}{2147483648} \quad \frac{1}{4294967296} \quad \frac{1}{8589934592} \quad \frac{1}{17179869184} \quad \frac{1}{34359738368} \quad \frac{1}{68719476736} \quad \frac{1}{137438953472} \quad \frac{1}{274877906944} \quad \frac{1}{549755813888} \quad \frac{1}{1099511627776} \quad \frac{1}{2199023255552} \quad \frac{1}{4398046511104} \quad \frac{1}{8796093022208} \quad \frac{1}{17592186044416} \quad \frac{1}{35184372088832} \quad \frac{1}{70368744177664} \quad \frac{1}{140737488355328} \quad \frac{1}{281474976710656} \quad \frac{1}{562949953421312} \quad \frac{1}{1125899906842624} \quad \frac{1}{2251799813685248} \quad \frac{1}{4503599627370496} \quad \frac{1}{9007199254740992} \quad \frac{1}{18014398509481984} \quad \frac{1}{36028797018963968} \quad \frac{1}{72057594037927936} \quad \frac{1}{144115188075855872} \quad \frac{1}{288230376151711744} \quad \frac{1}{576460752303423488} \quad \frac{1}{1152921504606846976} \quad \frac{1}{2305843009213693952} \quad \frac{1}{4611686018427387904} \quad \frac{1}{9223372036854775808} \quad \frac{1}{18446744073709551616} \quad \frac{1}{36893488147419103232} \quad \frac{1}{73786976294838206464} \quad \frac{1}{147573952589676412928} \quad \frac{1}{295147905179352825856} \quad \frac{1}{590295810358705651712} \quad \frac{1}{1180591620717411303424} \quad \frac{1}{2361183241434822606848} \quad \frac{1}{4722366482869645213696} \quad \frac{1}{9444732965739290427392} \quad \frac{1}{18889465931478580854784} \quad \frac{1}{37778931862957161709568} \quad \frac{1}{75557863725914323419136} \quad \frac{1}{151115727451828646838272} \quad \frac{1}{302231454903657293676544} \quad \frac{1}{604462909807314587353088} \quad \frac{1}{1208925819614629174706176} \quad \frac{1}{2417851639229258349412352} \quad \frac{1}{4835703278458516698824704} \quad \frac{1}{9671406556917033397649408} \quad \frac{1}{19342813113834066795298816} \quad \frac{1}{38685626227668133590597632} \quad \frac{1}{77371252455336267181195264} \quad \frac{1}{154742504910672534362390528} \quad \frac{1}{309485009821345068724781056} \quad \frac{1}{618970019642690137449562112} \quad \frac{1}{1237940039285380274899124224} \quad \frac{1}{2475880078570760549798248448} \quad \frac{1}{4951760157141521099596496896} \quad \frac{1}{9903520314283042199192993792} \quad \frac{1}{19807040628566084398385987584} \quad \frac{1}{39614081257132168796771975168} \quad \frac{1}{79228162514264337593543950336} \quad \frac{1}{158456325028528675187087900672} \quad \frac{1}{316912650057057350374175801344} \quad \frac{1}{633825300114114700748351602688} \quad \frac{1}{1267650600228229401496703205376} \quad \frac{1}{2535301200456458802993406410752} \quad \frac{1}{5070602400912917605986812821504} \quad \frac{1}{10141204801825835211973625643008} \quad \frac{1}{20282409603651670423947251286016} \quad \frac{1}{40564819207303340847894502572032} \quad \frac{1}{81129638414606681695789005144064} \quad \frac{1}{162259276829213363391578010288128} \quad \frac{1}{324518553658426726783156020576256} \quad \frac{1}{649037107316853453566312041152512} \quad \frac{1}{1298074214633706907132624082305024} \quad \frac{1}{2596148429267413814265248164610048} \quad \frac{1}{5192296858534827628530496329220096} \quad \frac{1}{10384593717069655257060992658440192} \quad \frac{1}{20769187434139310514121985316880384} \quad \frac{1}{41538374868278621028243970633760768} \quad \frac{1}{83076749736557242056487941267521536} \quad \frac{1}{166153499473114484112975882535043072} \quad \frac{1}{332306998946228968225951765070086144} \quad \frac{1}{664613997892457936451903530140172288} \quad \frac{1}{1329227995784915872903807060280344576} \quad \frac{1}{2658455991569831745807614120560689152} \quad \frac{1}{5316911983139663491615228241121378304} \quad \frac{1}{10633823966279326983230456482242756608} \quad \frac{1}{21267647932558653966460912964485513216} \quad \frac{1}{42535295865117307932921825928971026432} \quad \frac{1}{85070591730234615865843651857942052864} \quad \frac{1}{170141183460469231731687303715884105728} \quad \frac{1}{340282366920938463463374607431768211456} \quad \frac{1}{680564733841876926926749214863536422912} \quad \frac{1}{1361129467683753853853498429727072845824} \quad \frac{1}{2722258935367507707706996859454145691648} \quad \frac{1}{5444517870735015415413993718908291383296} \quad \frac{1}{108890357414700308$

$$a_{n+1} = a_n \cdot q$$

$$q = \frac{a_{n+1}}{a_n}$$

$$a_m = a_1 \cdot q^{m-1}$$

$$a_n = a_k \cdot q^{n-k}$$

$\xleftrightarrow{\text{TAB}}$

$$a_{n+1} = a_n + r$$

$$r = a_{n+1} - a_n$$

$$a_m = a_1 + (m-1) \cdot r$$

$$a_m = a_k + (m-k) \cdot r$$

$a_{n+1}, a_n, a_{n-1}, \dots, a_1$

$(a_n)^2 = a_{n-1} \cdot a_{n+1}$

$a_{n-k-1}, a_{n-1}, a_{n+k} \quad n, k \in \mathbb{N}^+$

$a_m^2 = a_{n-k} \cdot a_{n+k}$

$a_{10}^2 = a_5 \cdot a_{17}$

$a_8 = a_{10} \cdot q^{-2}$

$$a_{17} = a_{10} \cdot q^7$$

b) $\frac{2+\sqrt{3}}{2-\sqrt{3}}, \frac{2+\sqrt{3}}{2}, \frac{1}{4}$

$$\rho = \frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} \cdot \frac{1}{4} = \frac{(2+\sqrt{3})^2}{4} = 1$$

$$\underbrace{a_{m-1}}_1 = \underbrace{a_m \cdot \frac{1}{q}}_1, \underbrace{a_{m+1}}_1 = \underbrace{a_m \cdot q}_1, \underbrace{a_{m+2}}_1 = \underbrace{a_m \cdot q^2}_1, \dots$$

$$a_{m+1} = a_m \cdot q$$
$$q = \frac{a_{m+1}}{a_m} = \frac{a_m}{a_{m-1}} \Leftrightarrow (a_m)^2 = a_{m-1} \cdot a_{m+1}$$
$$a_n = \sqrt{a_{n-1} \cdot a_{n+1}} \vee a_n = -\sqrt{a_{n-1} \cdot a_{n+1}}$$

$$a_n = a_1 \cdot q^{n-1}$$

$$a_2 = a_1 \cdot q$$

$$a_3 = a_2 \cdot q = a_1 \cdot q \cdot q = a_1 \cdot q^2$$

$$a_4 = a_3 \cdot q = a_1 \cdot q^2 \cdot q = a_1 \cdot q^3$$

7.66. Wyznacz pierwszy wyraz i iloraz ciągu geometrycznego (a_n) , wiedząc, że:

- a) $a_5 - a_3 = 1680$ i $a_3 + a_4 = 560$
 b) $a_6 - a_4 = 432$ i $a_5 - a_4 = 108$
 c) $a_7 - a_3 = 120$ i $a_7 - a_5 = 96$
 d) $a_1 + a_5 = 1285$ i $a_2 \cdot a_4 = 6400$.

$$a_5 = a_1 \cdot q^4$$

$$a_6 = a_1 \cdot q^5$$

$$a_4 = a_1 \cdot q^3$$

$$\begin{cases} a_1 \cdot q^4 - a_1 \cdot q^2 = 1680 \\ a_1 \cdot q^3 + a_1 \cdot q^4 = 560 \end{cases}$$

$$\begin{cases} a_1 \cdot q^2 (q^2 - 1) = 1680 \\ a_1 \cdot q^3 (q + 1) = 560 \end{cases}$$

$$a_1 \cdot q^2 \cdot 5 = 560$$

$$a_1 = \frac{560}{5} = 112$$

$$\frac{a_1 \cdot q^2 (q^2 - 1)}{a_1 \cdot q^3 (q + 1)} = \frac{1680}{560} = 3$$

$$q - 1 = 3$$

$$q = 4$$

7.66. Wyznacz pierwszy wyraz i iloraz ciągu geometrycznego (a_n) , wiedząc, że:

- a) $a_5 - a_3 = 1680$ i $a_3 + a_4 = 560$
 b) $a_6 - a_4 = 432$ i $a_5 - a_4 = 108$
 c) $a_7 - a_3 = 120$ i $a_7 - a_5 = 96$
 d) $a_1 + a_5 = 1285$ i $a_2 \cdot a_4 = 6400$.

$$a_2 = a_1 \cdot q$$

$$a_5 = a_1 \cdot q^4$$

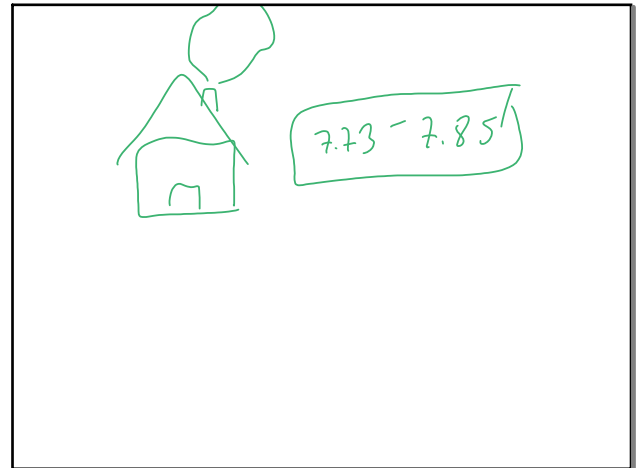
$$\begin{cases} a_1 \cdot q^4 - a_1 \cdot q^2 = 1680 \\ a_1 \cdot q^3 + a_1 \cdot q^4 = 560 \end{cases}$$

$$\frac{q^2 + 1}{q^2} = \frac{1680}{560} = \frac{3}{1}$$

$$q^2 + 1 = 3q^2 \Rightarrow 2q^2 = 1 \Rightarrow q^2 = \frac{1}{2} \Rightarrow q = \pm \frac{1}{\sqrt{2}}$$

$$q^2 - 4 = 0$$

$$q = 2 \vee q = -2$$



7.89. Uzupełnij tabelkę brakującymi danymi dotyczącymi ciągu geometrycznego (a_n) .

a_1	q	a_n	n	S_n
9		144	5	
	2		9	4088
	$\frac{1}{2}$	$\frac{5}{16}$		$19\frac{11}{16}$
4	3			13 120
	6		5	1555
2		1458		2186
	$\sqrt{5}$	$625\sqrt{5}$	8	
$-\sqrt{3}$	$\sqrt{6}$	$-108\sqrt{2}$		