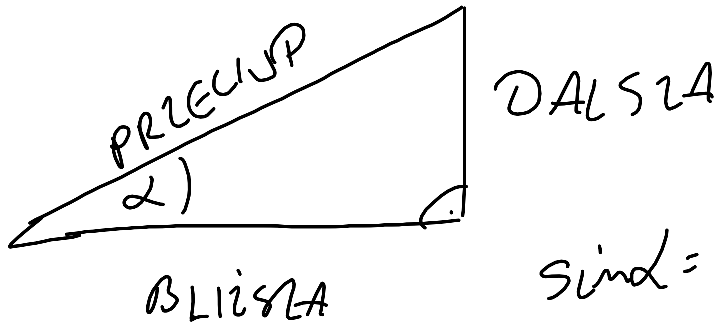
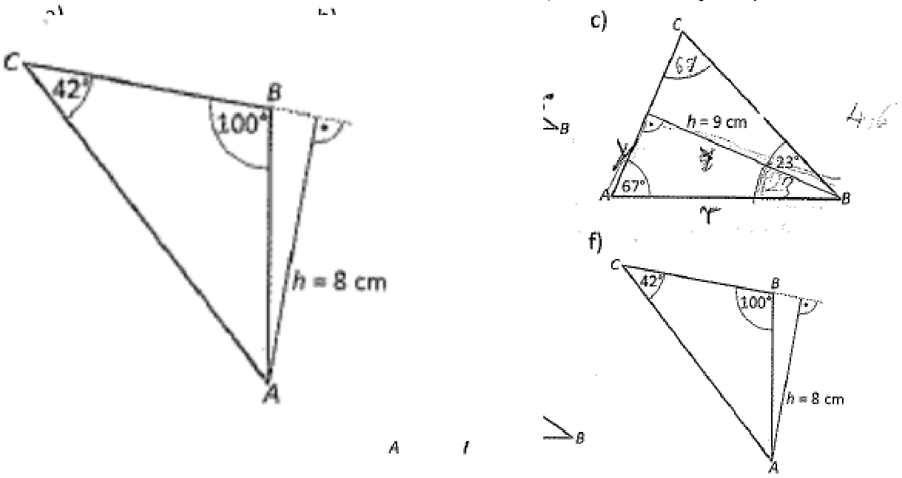
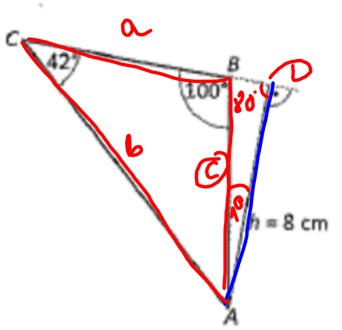


$\sin\alpha = \dfrac{\text{DALSZA}}{\text{PRZECIW.}}$

$\cos\alpha =$

6.4. **6.4.** Oblicz obwód trójkąta *ABC* na rysunku poniżej z dokładnością do 0,5 cm:





$AB = c$ ✓

$BC = a$

$CA = b$ ✓

2 △ABD $\sin 80° = \frac{8}{c} \Rightarrow c = \frac{8}{\sin 80°} = 8{,}12$

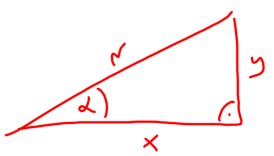
$\sin 80° \approx 0{,}98$

△CAD $\Rightarrow \sin 42° = \frac{8}{b} \Rightarrow b = \frac{8}{\sin 42} = 11{,}95$

$\sin 42 \approx 0{,}669$

$a = CD - BD =$

$\operatorname{tg} 42° = \frac{8}{CD} \Rightarrow CD = \frac{8}{\operatorname{tg} 42°} = \frac{8}{0{,}9} = 8{,}88$



$\sin\alpha = \frac{y}{r}$

$\cos\alpha = \frac{x}{r}$

$\operatorname{tg}\alpha = \frac{y}{x}$

$\operatorname{ctg}\alpha = \frac{x}{y}$

$x, y, r \in \mathbb{R}_+$

$x^2 + y^2 = r^2 \quad / : r^2$

$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$

$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$

$\alpha \neq \frac{k\pi}{2}$

$\boxed{\operatorname{tg}\alpha \cdot \operatorname{ctg}\alpha = 1}$ — 2o4

$\boxed{\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha}}$ $\boxed{\operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha}}$

$\alpha \neq \frac{\pi}{2} + k\pi$ $\alpha \neq k\pi$ $\alpha \neq \pi + k\pi$

$$\boxed{\begin{aligned} &\sin^2\alpha + \cos^2\alpha = 1 \\ &1 - \sin^2\alpha = \cos^2\alpha \\ &1 - \cos^2\alpha = \sin^2\alpha \end{aligned}}$$

6.56. **6.81.** Sprawdź, czy podane równości są tożsamościami trygonometrycznymi, wiedząc, że $\alpha \in (0°, 90°) \cup (90°, 180°)$:

a) $1-2\sin^2\alpha = 2\cos^2\alpha - 1$ b) $\cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1$

c) $\sin\alpha \cdot \left(\frac{1}{\sin\alpha} - \sin\alpha\right) = \cos^2\alpha$ d) $\cos\alpha \cdot \left(\frac{1}{\cos\alpha} - \cos\alpha\right) = \sin^2\alpha$

e) $\frac{2}{\cos^2\alpha} - 1 = 1 + 2\mathrm{tg}^2\alpha$ f) $\frac{2}{\sin^2\alpha} - 1 = 1 + 2\mathrm{ctg}^2\alpha$

g) $\frac{1}{1-\cos\alpha} + \frac{1}{1+\cos\alpha} = \frac{2}{\sin^2\alpha}$ h) $\frac{1}{1-\sin\alpha} + \frac{1}{1+\sin\alpha} = \frac{2}{\cos^2\alpha}$

i) $1-\sin\alpha = \frac{\mathrm{ctg}\,\alpha - \cos\alpha}{\mathrm{ctg}\,\alpha}$ j) $1-\cos\alpha = \frac{\mathrm{tg}\,\alpha - \sin\alpha}{\mathrm{tg}\,\alpha}$

$180°$ $\downarrow$ $\boxed{\pi}$ $270°$ $\frac{3}{2}\pi$

$\frac{3\pi}{2}$ $\frac{\pi}{2}$

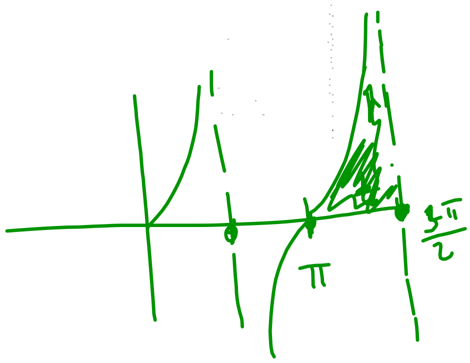
$270° \leftarrow 90°$

6.82. Wykaż, że jeśli $\alpha \in (180°, 270°)$, to $\frac{\sqrt{1-\cos^2\alpha}}{1+\cos\alpha} + \frac{1+\sqrt{1-\sin^2\alpha}}{\sin\alpha} = 0.$

6.83. Wykaż, że jeśli $\alpha \in (270°, 360°)$,

to $\frac{1-\sqrt{1-\cos^2\alpha}}{\cos\alpha} + \frac{\sqrt{1-\sin^2\alpha}}{1-\sin\alpha} = \frac{2\cos\alpha}{1-\sin\alpha}.$

$\mathrm{tg} \lessgtr \alpha$



6.83. Wykaż, że jeśli $\alpha \in (270°, 360°)$, $\left(\frac{3\pi}{2}\ 2\pi\right)$

to $\frac{1-\sqrt{1-\cos^2\alpha}}{\underset{+}{\cos\alpha}} + \frac{\sqrt{1-\sin^2\alpha}}{\underset{+}{1-\sin\alpha}} = \frac{2\cos\alpha}{\underset{+}{1-\sin\alpha}}.$

2 : . . .

$\sqrt{a^2} = |a|$

$$L = \frac{1-\sqrt{\sin^2\alpha}}{\cos\alpha} + \frac{\sqrt{\cos^2\alpha}}{1-\sin\alpha} =$$

$$\frac{1-|\overset{-}{\sin\alpha}|}{\cos\alpha} + \frac{|\overset{+}{\cos\alpha}|}{1-\sin\alpha} = \frac{1+\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{1-\sin\alpha} =$$

$$= \frac{1-\sin^2\alpha + \cos^2\alpha}{\cos\alpha(1-\sin\alpha)} = \frac{2\cos^{\cancel{2}}\alpha}{\cancel{\cos\alpha}(1-\sin\alpha)} = P$$

g) $\frac{1}{1-\cos\alpha}+\frac{1}{1+\cos\alpha}=\frac{2}{\sin^2\alpha}$

$\boxed{\alpha \neq -\pi + k\pi}$

$L = P$

ZAł: $\sin\alpha \neq 0$

$\cos\alpha \neq 1$

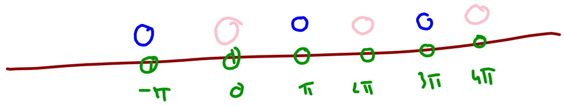
$\cos\alpha \neq -1$

$k \in \mathbb{Z}$

$\alpha \neq 0 + 2k\pi$

$\alpha \neq -\pi + 2k\pi$

$\alpha \neq -\pi + k\pi$



g) $\frac{1}{1-\cos\alpha}+\frac{1}{1+\cos\alpha}=\frac{2}{\sin^2\alpha}$

$L = \frac{1}{1-\cos\alpha} + \frac{1}{1+\cos\alpha} = \frac{1(1+\cos\alpha) + 1-\cos\alpha}{(1-\cos\alpha)(1+\cos\alpha)} =$

$\frac{2}{1-\cos^2\alpha} = \frac{2}{\sin^2\alpha} = P$ c n d