

Zadanie 48.

Dana jest funkcja f postaci

$$f(x) = \sin 2x - \cos x + 2(\bar{a} - 1) \sin x - a + 1.$$

Wyznacz zbiór wartości parametru a , dla których rozwiązaniami równania $f(x) = 0$ są tylko liczby postaci $\frac{\pi}{6} + 2k\pi$ lub $\frac{5\pi}{6} + 2k\pi$, gdzie $k \in \mathbb{C}$.

$$\text{i)} \quad 2\sin x \cos x - \cos^2 x + 2(\bar{a} - 1) \sin x - a + 1 = 0$$

$$\text{ii)} \quad 2 \sin(\cos x + |\bar{a}| - 1) = (\cos x + a - 1) = 0$$

$$\begin{cases} 2\sin x \cos x - \cos^2 x = 0 \\ \cos x + a - 1 = 0 \end{cases} \Rightarrow \begin{cases} \cos x = 0 \\ \cos x = -a + 1 \end{cases}$$

Zadanie 48.

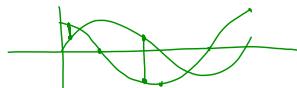
Dana jest funkcja f postaci

$$f(x) = \sin 2x - \cos x + 2(\bar{a} - 1) \sin x - a + 1.$$

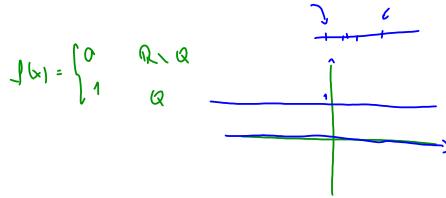
Wyznacz zbiór wartości parametru a , dla których rozwiązaniami równania $f(x) = 0$ sątylko liczby postaci $\frac{\pi}{6} + 2k\pi$ lub $\frac{5\pi}{6} + 2k\pi$, gdzie $k \in \mathbb{C}$.

$$x = \frac{\pi}{6} + 2k\pi \Rightarrow \sin^2 x - \cos^2 x + 2(\bar{a} - 1) \cdot \frac{1}{2} - a + 1 = 0$$

$$|\bar{a}| = a \Rightarrow a > 0 \Rightarrow a > 0$$

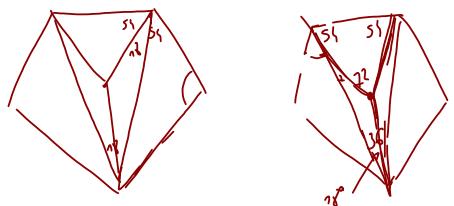


| Nr | Imię i nazwisko | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | (w sumie) | (w normali) | |
|-----|-----------------------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|-----------|-------------|-----|
| 1. | Bartosz Mordarski | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% | |
| 2. | Mikołaj Leonhardt | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 3. | Marcin Ziółkowski | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 4. | Jakub Adamczyk | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 5. | Krystian Góra Katarzyna Zuchowska | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 6. | Aleksandra Wiczorek | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 7. | Sławek Słapski | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 8. | Marta Figurska | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 9. | Martyna Wójcik | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 10. | Aleksandra Wiczorek | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 11. | Wojciech Popławski | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 12. | Bartosz Rzepecki | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 13. | Przemysław Ciecielowicz | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 14. | Jakub Zieliński | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 15. | Jakub Pisarek | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 16. | Wojciech Krysiak | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 17. | Paweł Bielaś | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 18. | Patrycja Lewandowska | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 19. | Honorata Kucharska | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 20. | Jakub Mędrak | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 21. | Natalia Mędrak | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 22. | Martyna Skierkowska | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 23. | Szymon Skierkowski | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 24. | Kacper Stenius | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |
| 25. | Kacper Stenius | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 75 | 48% |



$$\begin{aligned} \text{Zad. } \text{i)} \quad & f(x) \geq 0 \quad \text{dla } x \in [0, \pi] \\ \text{Zad. ii)} \quad & x_1 - x_2 < 0 \Rightarrow f(x_1) - f(x_2) < 0 \\ & \text{Zad. iii)} \quad f(x_1) - f(x_2) = a_1 - a_2 \Rightarrow a_1 - a_2 < 0 \Rightarrow a_1 < a_2 \\ \text{Zad. iv)} \quad & y = \sqrt{3-x_1+2} \\ & x_1 - x_2 < 0, \quad f(x) = \sqrt{3-x+2}, \quad x, x_2 \in (-\infty, 3) \\ & f(x_1) - f(x_2) = \sqrt{3-x_1+2} - \sqrt{3-x_2+2} \\ & \frac{(\sqrt{3-x_1+2} - \sqrt{3-x_2+2})(\sqrt{3-x_1+2} + \sqrt{3-x_2+2})}{\sqrt{3-x_1+2} + \sqrt{3-x_2+2}} = \frac{3-x_1+2 - 3+x_2-2}{\sqrt{3-x_1+2} + \sqrt{3-x_2+2}} = \frac{x_2 - x_1}{\sqrt{3-x_1+2} + \sqrt{3-x_2+2}} > 0 \Rightarrow x_2 - x_1 > 0 \end{aligned}$$

$$\begin{aligned} \text{Zad. vi)} \quad & \text{D: } x \in \mathbb{R} \setminus \{0\} \\ & \text{Dla } x > 0 \quad \sin x > 0 \quad \cos x > 0 \\ & \text{Dla } x < 0 \quad \sin x < 0 \quad \cos x < 0 \\ & \text{Dla } x = 0 \quad \sin x = 0 \quad \cos x = 1 \\ & \text{Dla } x = \pi \quad \sin x = 0 \quad \cos x = -1 \\ & \text{Dla } x = 2\pi \quad \sin x = 0 \quad \cos x = 1 \\ & \text{Dla } x = \pi/2 \quad \sin x = 1 \quad \cos x = 0 \\ & \text{Dla } x = 3\pi/2 \quad \sin x = -1 \quad \cos x = 0 \\ & \text{Dla } x = 5\pi/2 \quad \sin x = 1 \quad \cos x = 0 \end{aligned}$$



Zadanie 51.

Wyznacz zbiór wszystkich wartości parametru $a \in \mathbb{R}$, dla których równania
 $\sin 3x + \cos 2x = 1 + 2 \sin x \cos 2x$ i $\sin 3x = a \sin x + (4 - 2|a|) \sin^2 x$
są równoważne?

Zadanie 54.

Udowodnij, że jeśli x i y są liczbami z przedziału $(0; \pi)$ spełniającymi warunek $x + y < \pi$,
to zachodzi nierówność

$$\operatorname{ctg} x + \operatorname{ctg} y \geq 2 \operatorname{ctg} \frac{x+y}{2}.$$