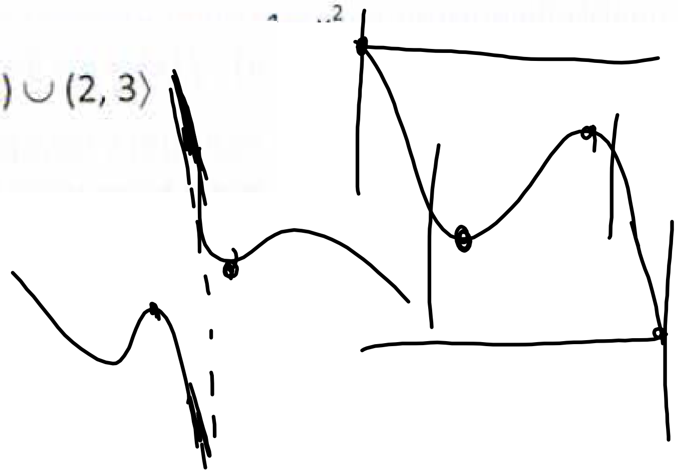
2.105. Wyznacz wartości (o ile istnieją) funkcji f: największą (M) i najmniejszą (m) w podanym zbiorze:

d) $f(x)=\dfrac{1}{x^2-4}, x \in \langle 1, 2) \cup (2, 3\rangle$



d) $f(x)=\frac{1}{x^2-4}$, $x \in \langle 1, 2) \cup (2, 3\rangle$

$D: x \in \mathbb{R} \setminus \{-2, 2\} \Rightarrow$

$D: \langle 1, 2) \cup (2, 3\rangle$

$$f'(x) = \frac{-2x}{(x^2-4)^2}$$

WK: $f'(x)=0 \Leftrightarrow x=0 \notin D$

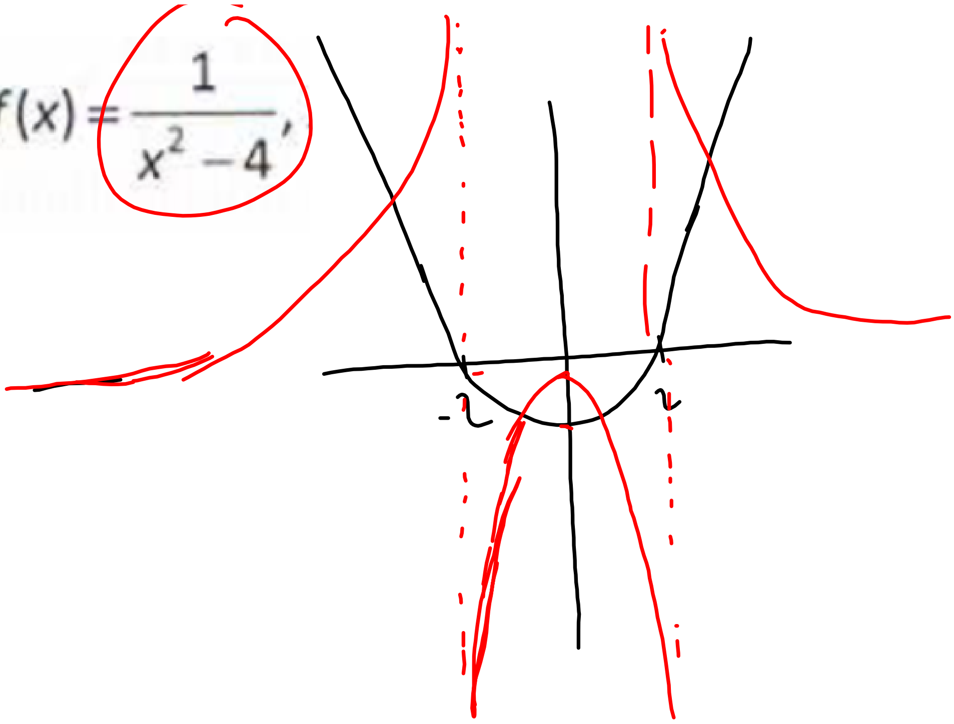
WW $\operatorname{sgn} f'(x)$:

x	$(-\infty, 0)$	0	$(0, +\infty)$
$f'(x)$	$+$	0	$-$
~~$f(x)$~~			

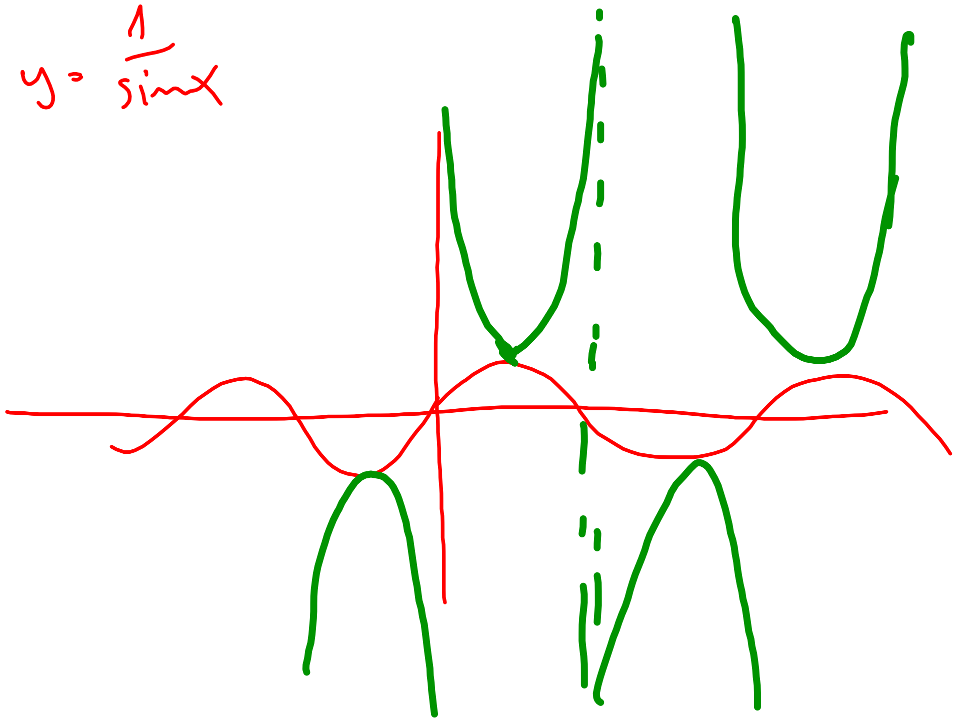
$x \in$	1	$(1, 2)$	2	$(2, 3)$	3
f'	$-$	$-$	X	$-$	$-$
f	$-\frac{1}{3}$	$\searrow -\infty$	as pion $x=2$	$+\infty \searrow$	$\frac{1}{5}$

ZW: $y \in (-\infty, -\frac{1}{3}\rangle \cup \langle \frac{1}{5}, +\infty)$

$$f(x)=\frac{1}{x^2-4},$$



$y = \frac{1}{\sin x}$



BADANIE P.Z.F.

c) $f(x)=\dfrac{x^3}{x^2-4}$

1) $D_f: x\in(-\infty,-2)\cup(-2,2)\cup(2,+\infty)$

2) $L(-2)=\lim\limits_{x\to-2^-}\dfrac{x^3}{x^2-4}=\left[\dfrac{-8}{0^+}\right]=-\infty$

MIAN

$(x-2)(x+2)$

$-4 \quad 0^-$

2b) $P(-2)=\lim\limits_{x\to-2^+} f(x)=\left[\dfrac{-8}{0^-}\right]=+\infty$

as pion $x=-2$

2c) $L(2)=\lim\limits_{x\to2^-}\dfrac{x^3}{x^2-4}=\left[\dfrac{8}{0^-}\right]=-\infty$

$P(2)=\lim\limits_{x\to2^+} f(x)=\left(\dfrac{8}{0^+}\right)=+\infty$

as pion $x=2$

$\lim\limits_{x\to-\infty}\dfrac{x^3}{x^2-4}=\lim\limits_{x\to-\infty}\dfrac{x^3}{x^2(1-\frac{4}{x^2})}=-\infty$

nie ma as poziomej

$\lim\limits_{x\to+\infty}\dfrac{x^3}{x^2-4}=+\infty$

3) asympt. $\boxed{x=-2}$ $\boxed{x=2}$

$y=ax+b$

$a=\lim\limits_{x\to\pm\infty}\dfrac{x^3}{x^3(1-\frac{4}{x^2})}=1$

$b=\lim\limits_{x\to\pm\infty}(f(x)-ax)=\lim\limits_{x\to\pm\infty}\dfrac{\cancel{x^3}-\cancel{x^3}+4x}{x^2-4}=$

$\lim\limits_{x\to\pm\infty}\dfrac{4}{x(1-\frac{4}{x^2})}=0$

$\boxed{y=x}$

4) Punkty wspólne z osiami

OX: $(0,0)$ $\qquad \dfrac{x^3}{x^2-4}=0 \iff x=0$

OY: $(0,0)$

5) PARZYSTA NIEP.

1) N $f(-x)=\dfrac{(-x)^3}{(-x)^2-4}=-\dfrac{x^3}{x^2-4}=-f(x)$

F. jest nieparzysta wykres jest symetryczny względem $(0,0)$

II $f'(x)$ ....

x	
f'	
f	

$$f(x)=\frac{x^3}{x^2-4}$$

$\overline{\text{II}}$ $f'(x) = \dfrac{x^2(x^2-12)}{(x^2-4)^2}$

1) $D_{f'} = D_f = \mathbb{R}\setminus\{-2,2\}$

3) WK $f'(x)=0$

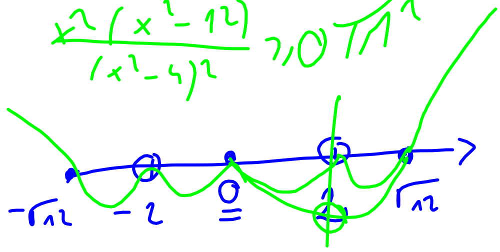
$\Updownarrow$

$x=0 \vee x=-\sqrt{12} \vee x=\sqrt{12}$

4) WW $\operatorname{sgn} f'(x)$

$\dfrac{x^2(x^2-12)}{(x^2-4)^2} \geq 0 \quad / \cdot ()^2$

$x^2(x^2-12)(x-2)^2(x+2)^2 \geq 0$

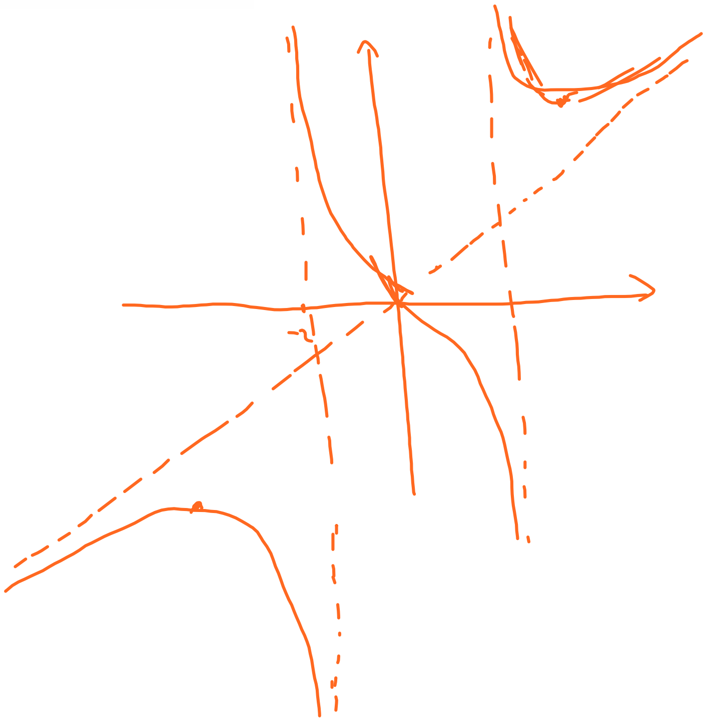


5) TABELKA

x	$(-\infty,-\sqrt{12})$	$-\sqrt{12}$	$(-\sqrt{12},-2)$	-2	$(-2,0)$	0	$(0,2)$	2	$(2,\sqrt{12})$	$\sqrt{12}$	$(\sqrt{12},+\infty)$
f'	+	0	−	X	−	0	−	X	−	0	+
f	$-\infty$ ↗	max $-3\sqrt{3}$	↘ $-\infty$	as pio	$+\infty$ ↘	0	↘ $-\infty$	as pio	$+\infty$ ↘	min $3\sqrt{3}$	↗ $+\infty$

c) $f(x)=\dfrac{x^3}{x^2-4}$

$f(-\sqrt{12}) = \dfrac{-\cancel{8}\cdot 3\sqrt{3}}{\cancel{12-4}} = -3\sqrt{3}$



2.110. Zbadaj przebieg zmienności i naszkicuj wykres funkcji f:

a) $f(x)=x-3+\frac{2x-6}{x+2}+\frac{4x-12}{(x+2)^2}+\dots$ b) $f(x)=x-2+\frac{x-2}{x-5}+\frac{x-2}{(x-5)^2}+\dots$

$f(x)=x-2+\frac{x-2}{x-5}+\frac{x-2}{(x-5)^2}+\dots$

$a_1=x-2$

$|q|=\left|\frac{1}{x-5}\right|<1 \quad \Big| \; |x-5|>0 \qquad 1<|x-5|$

$\boxed{|x-5|>1}$

$x-5>1 \vee x-5<-1$

$x>6 \vee x<4$

$D: x\in(-\infty, 4)\cup(6,+\infty)$

$f(x)=\frac{x-2}{1-\frac{1}{x-5}}=\frac{x^2-7x+10}{x-6}$

$\boxed{f(x)=\frac{x^2-7x+10}{x-6}} \qquad \frac{(x-2)(x-5)}{(x-6)}$

$\lim_{x\to 4^-} f(x)=1 \qquad \boxed{\lim_{x\to 6^+} f(x)=+\infty} \Rightarrow$ as pion.

$\lim_{x\to\pm\infty} f(x)=\pm\infty \qquad \boxed{a=\lim f(x)\cdot\frac{1}{x}}$

as.

$y=ax+b$

$a=\lim_{x\to\pm\infty}\frac{x^2-7x+10}{x^2-6x}=\lim_{x\to\pm\infty}\frac{1-\frac{7}{x}+\frac{10}{x^2}}{1-\frac{6}{x}}=1$ a

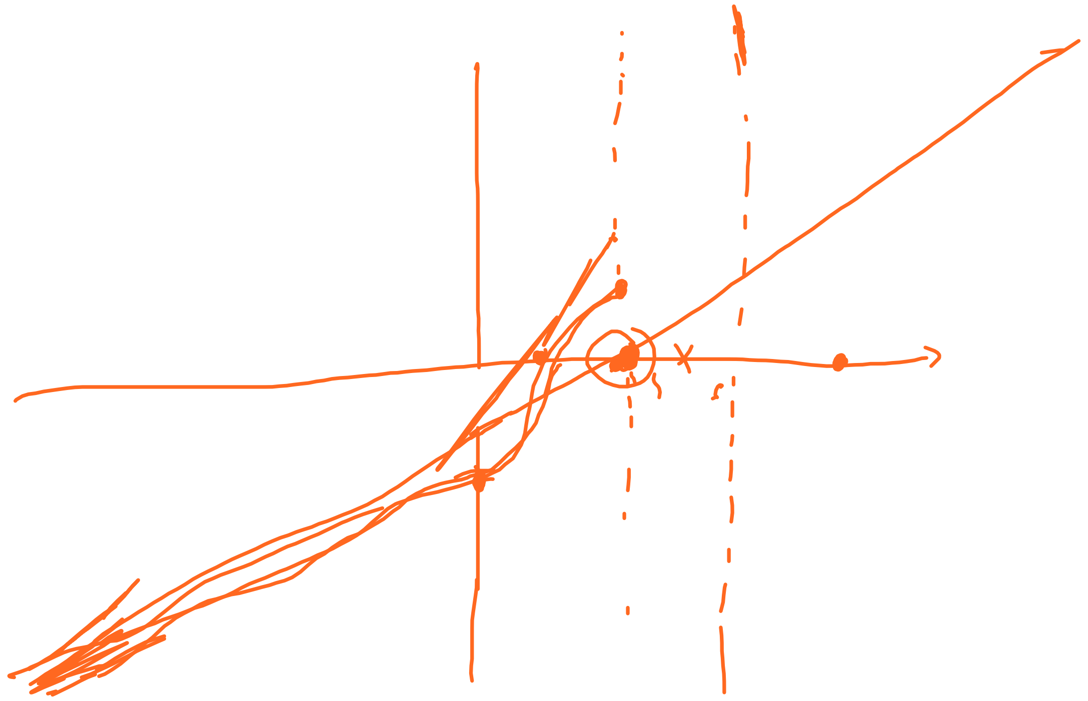
$b=\lim_{x\to\pm\infty}(f(x)-x)=\lim_{x\to\pm\infty}\left(\frac{x^2-7x+10-x^2+6x}{x-6}\right)$

$\lim_{x\to\infty}\frac{-x+10}{x-6}=-1 \qquad \boxed{y=x-1}$

$\boxed{f(x)=\frac{x^2-7x+10}{x-6}} \qquad \frac{(x-2)(x-5)}{(x-6)}$

OX: $(2,0)\ (5,0)$

OY: $\left(0,-\frac{5}{3}\right)$



$$f(x)=\frac{x^2-7x+10}{x-6}$$

$$\frac{(x-2)(x-5)}{(x-6)}$$

$$f'(x)=\frac{(2x-7)(x-6)-(x^2-7x+10)}{(x-6)^2}$$

$$f'(x)=\frac{2x^2-12x-7x+42-x^2+7x-10}{(x-6)^2}$$

$$f'(x)=\frac{x^2-12x+32}{(x-6)^2}=\frac{(x-8)(x-4)}{(x-6)^2}$$

$D_{f'}=D_f=(-\infty,4)\cup(6,+\infty)$

uk

$f'(x)=0 \Leftrightarrow x=4 \vee x=8$

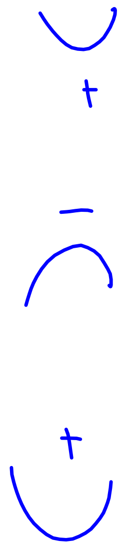
uu

$f'(x)>0$

5

x	$(-\infty,4)$	4	$(4,6)$	6	$(6,8)$	8	$(8,+\infty)$
f'	$+$	0	$-$	X	$-$	0	$+$
f	$-\infty$ ↗	1 max $(4,1)$	↘	as. pion. $+\infty$	↘	min lok 9 $(8,9)$	↗ $+\infty$

f''	$-$	$-$	$-$	0	$+$	$+$	$+$
f					min lok	



$$f'(x)=\frac{x^2-12x+32}{(x-6)^2}$$

$$f''(x)=\frac{(2x-12)(x-6)^2-2\cancel{(x-6)}(x^2-12x+32)}{(x-6)^{\cancel{4}\,3}}=$$

$$=\frac{2(\cancel{x^2}-\cancel{12x}+36-\cancel{x^2}+\cancel{12x}-32)}{(x-6)^3}=\frac{8}{(x-6)^3}$$

$f''\neq 0 \ \forall x$

f''