

2.192

$$V_{AB} = 70 \text{ km/h}$$

$$V_{BA} = 50 \text{ km/h}$$

$$S_{AB} = S_{BA} \quad \triangle$$

||

$$\bar{V} = \frac{2 \cdot 350}{350 \left(\frac{1}{70} + \frac{1}{50} \right)} = \frac{700}{5+7}$$

$$\frac{700}{12} = \underline{\underline{58\frac{1}{3} \text{ km/h}}}$$

2.193

$$V_1 = 72 \text{ km/h}$$

$$S_1 = S_2$$

$$\bar{V} = 57,6 \text{ km/h}$$

$$\bar{V} = \frac{2}{\frac{1}{V_1} + \frac{1}{V_2}} \quad | \cdot 1 : \bar{V}$$

$$V_2 = ?$$

$$\frac{1}{V_2} = \frac{2}{\bar{V}} - \frac{1}{V_1} = \frac{2V_1 - \bar{V}}{\bar{V} \cdot V_1} \quad \left| - \frac{1}{V_1} \right.$$

$$V_2 = \frac{\bar{V} \cdot V_1}{2V_1 - \bar{V}}$$

$$V_2 = 48 \text{ km/h}$$

$$\frac{1}{V_1} + \frac{1}{V_2} = \frac{2}{\bar{V}} \quad | - \frac{1}{V_1}$$

$$\frac{1}{V_2} = \frac{2}{\bar{V}} - \frac{1}{V_1}$$

4.2.3 indukcja

piątek, 3 grudnia 2021 13:27

4.2.3 indukcja $n \in \mathbb{N}$

$$T: 10^n + 4^n - 2 = 3l \quad l \in \mathbb{Z}$$

$$D: T(0) \div L(0) = 0 \quad 3|0 \quad OK.$$

$$Z_k: 10^k + 4^k - 2 = 3l_2$$

$$T_{k+1} \quad 10^{k+1} + 4^{k+1} - 2 = 3l_3$$

$$D: 10^{k+1} + 4^{k+1} - 2 = \underline{10 \cdot 10^k} + \underline{4 \cdot 4^k} - 2 =$$

$$\underline{10^k} + \underline{9 \cdot 10^k} + \underline{4^k} + \underline{3 \cdot 4^k} - 2 = \underbrace{3(l_2 + 3 \cdot 10^k + 4^k)}_{l_3 \in \mathbb{Z}}$$

Wi.

4.1.9

$$1^3 + 2^3 + 3^3 + \dots + m^3 = (1 + 2 + 3 + \dots + m)^2 = \frac{(m+1)^2 \cdot m^1}{4}$$

$$(1 + 2 + 3 + \dots + m)^2 = \left(\frac{(m+1) \cdot m}{2} \right)^2$$

$$L(1) = 1 = P(1) = 1 \quad \text{OK}$$

$$Z_k: 1^3 + 2^3 + \dots + k^3 = (1 + 2 + \dots + k)^2$$

$$T_{k+1}: \underbrace{1^3 + \dots + k^3}_{Z_k} + (k+1)^3 = \underbrace{(1 + \dots + k + (k+1))}_{a+b}^2 = \underbrace{(1 + 2 + \dots + k)^2}_{()^2} + \underbrace{2(1 + \dots + k)(k+1)}_{+(k+1)^2}$$

$$D: \underbrace{(1 + 2 + \dots + k)^2}_{()^2} + \underbrace{(k+1)^3}_{(k+1)^3}$$

$$P_T = ()^2 + (k+1) \left(2 \underbrace{(1 + 2 + \dots + k)}_{(1+k)k/2} + k+1 \right) =$$

$$= (k+1) \left(2 \cdot \frac{(1+k)k}{2} + k+1 \right)$$

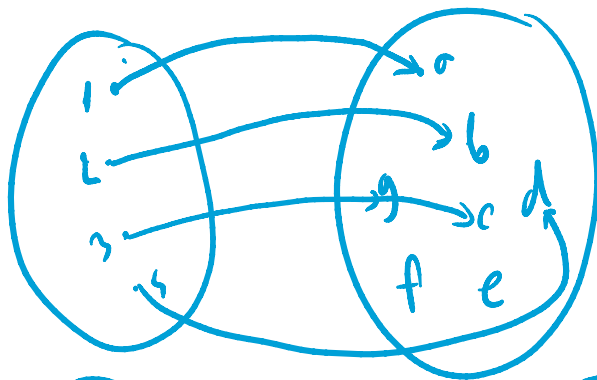
$$k + k^2 + k + 1 = k^2 + 2k + 1$$

$$1 + 2 + 3 + \dots + n$$

$$n + n-1 + n-2 \dots + 2 + 1$$

$$\frac{n \cdot (n+1)}{2}$$

$$(n+1) + (n+1)$$



$$\cancel{f(2)=6}$$

$$\cancel{f(2)=8}$$

$x \in \mathbb{D}$
 $\uparrow$
 x - argumenty

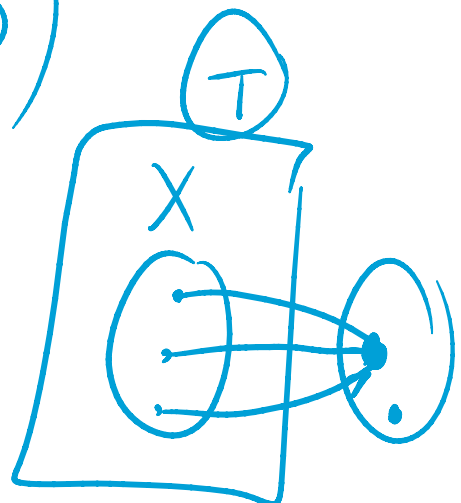
$y \in \mathbb{D}$
 $\uparrow$
 wartości f-ty.

X (drugi - argument $\mathbb{D}$)

Y (

f:

N ???



$$f(x) = x^2$$

f od -1

$$f(-1) = (-1)^2 = 1$$

$$f(1) = 1^2 = 1$$

x	-1	1	0
y	1	1	0

$$\{(-1, 1); (1, 1); (0, 0); (5, 25) \dots\}$$

argumenty wartości

$$y = f(x)$$

$\uparrow$
 wartości = f od x

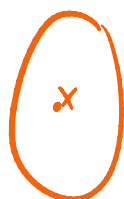
LAURELA

LAURELA

...

$$\overset{\text{ZAWIERA}}{\mathbb{D}} \subset \mathbb{R} \wedge \overset{\text{ZAWIERA}}{\mathbb{I}} \subset \mathbb{R} \Rightarrow \underline{f. \text{ liniowa}}$$

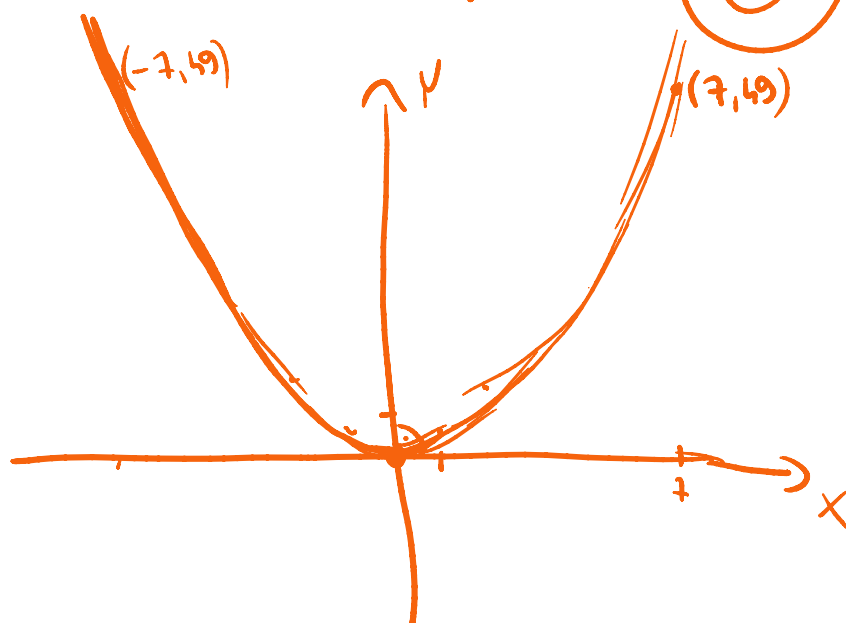
$$\boxed{x \in \mathbb{D}}$$



$$K \in \mathbb{ID}$$



$$\text{kobiety} \subset \mathbb{ID}$$



$$\begin{matrix} (x, f(x)) \\ \uparrow \quad \uparrow \end{matrix}$$

$$\boxed{f(x) = x^2}$$

$$\boxed{\mathbb{D}: x \in \mathbb{R}}$$

$$\begin{aligned} f(7) &= 7^2 = 49 \quad (7, 49) \\ f(-7) &= (-7)^2 = 49 \quad (-7, 49) \end{aligned}$$

$$f(0) = 0^2 = 0 \quad (0, 0)$$

$$f(1) = 1^2 = 1 \quad (1, 1)$$

$$f(-\frac{1}{2}) = \frac{1}{4} = (-\frac{1}{2}, \frac{1}{4})$$

3.19 abcd

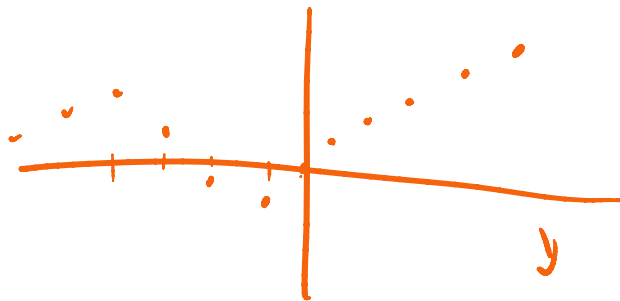
$$(g(11))^{\frac{1}{g(7)-g(5)}}$$

$$g(x) = x - 2$$

$$\textcircled{II}: x \in \{2, 3, \dots, 13\} \cap \mathbb{P}$$

$$g^{\frac{1}{5-3}} = g^{\frac{1}{2}} = \underline{3}$$

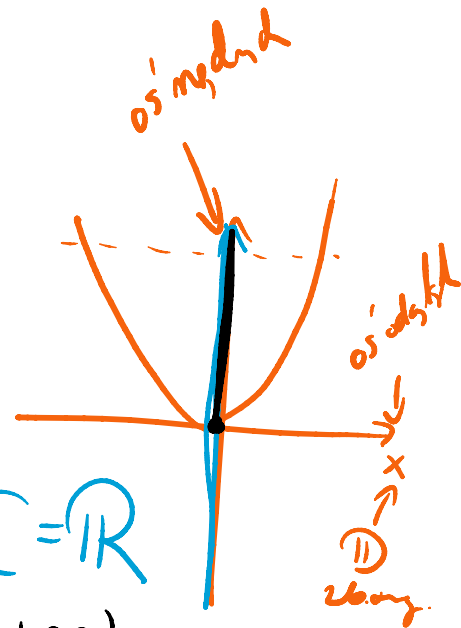
$$\textcircled{II}: x \in \mathbb{Z}$$



$$\mathbb{Z} \cup \mathbb{E} \subseteq \textcircled{II}$$

$$\textcircled{II} = \mathbb{R}$$

$$\mathbb{Z} \cup \{y \in (0, +\infty)\}$$



$$f: X \rightarrow Y$$

$$f: X \rightarrow Y$$

$$f(x) = \sqrt{x} \quad \underbrace{x \in \mathbb{R}}$$

$$\text{zak: } x \geq 0$$

$$\text{D: } x \in (0, +\infty)$$

$$f(x) = \log_a x \leftarrow \text{funkcja logarytmiczna}$$

$\uparrow$ argument $\uparrow$ podstawa

$$a > 0, a \neq 1$$

$$\text{D: } x \in \mathbb{R}_+$$

~~$f(x) = \log x$~~

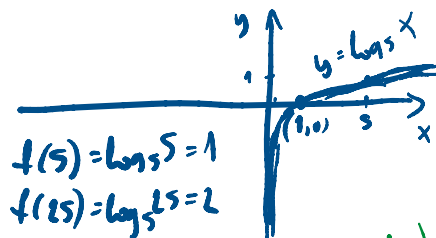
~~$f(x) = \log_1 x$~~

$f(x) = \log_5 x$

$$\text{D: } x \in \mathbb{R}_+$$

$$f(1) = \log_5 1 = 0$$

(1, 0)



ASYMPTOTA
PIONOWA (przechodzi do $-\infty$ gdy $x \rightarrow 0^+$)

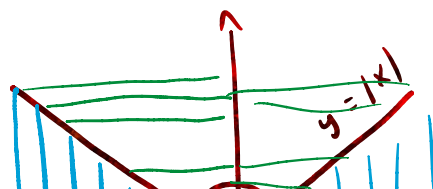
$$f(x) = \frac{a}{x}$$

$$\text{D: } x \in \mathbb{R} \setminus \{0\}$$

$$f(x) = \frac{a}{\sqrt{x}}$$

$$\text{D: } x \in \mathbb{R}_+$$

$$f(x) = y$$

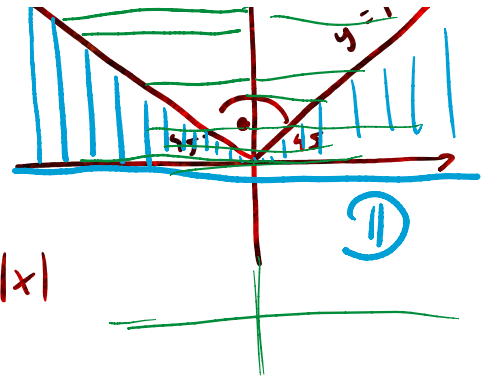


$$f(x) = \overbrace{|x|}^{\mathbb{R}}$$

$\mathbb{D}: x \in \mathbb{R}$

$$\text{Z.W.: } y \in \langle 0, +\infty \rangle$$

$$f(x) = |x|$$



3.53 d.f.

$$d) f(x) = \frac{\sqrt{x+4}}{\sqrt{x-3}} + \frac{x+2}{x^2-12x+36}$$

$$\text{Zau: } \underbrace{x+4 \geq 0}_{x \geq -4} \wedge \underbrace{x-3 > 0}_{\uparrow x > 3} \wedge \underbrace{(x-6)^2 \neq 0}_{x \neq 6}$$

$$\textcircled{II}: x \in (3, +\infty) \setminus \{6\}$$

$$f) f(x) = \frac{\sqrt{x^2+2x+1}}{x^2-4} - \frac{x^2+1}{\sqrt{7-2x}} = \frac{\sqrt{(x+1)^2}}{(x+2)(x-2)} - \frac{x^2+1}{\sqrt{7-2x}} = \frac{|x+1|}{(x+2)(x-2)} - \frac{x^2+1}{\sqrt{7-2x}}$$

$$\text{Zau: } \underbrace{(x+2)(x-2) \neq 0}_{x \neq -2 \wedge x \neq 2} \wedge \underbrace{7-2x > 0}_{x < 3.5}$$

ultraa.

$$\textcircled{II}: x \in (-\infty, 3.5) \setminus \{-2, 2\}$$

$$\sim (x=2 \vee x=-2) \\ (x \neq 2 \wedge x \neq -2)$$

3. 54 abcd x-między
a-punkt

$$f(x) = \frac{2x}{(x-a)(x+5)} \leftarrow$$

$$\begin{aligned} \text{Za: } (x-a)(x+5) &\neq 0 \\ x-a &\neq 0 \wedge x \neq -5 \\ x &\neq a \wedge x \neq -5 \end{aligned}$$

$$D: \mathbb{R} \setminus \{-5; a\}$$

$$\textcircled{D}: x \in \mathbb{R} \setminus \{-5; -2\}$$

$$\boxed{a = -2} \text{ def.}$$

b) $f(x) = \frac{5}{x^2 + a}$

$$\textcircled{II}: x \in \mathbb{R} \setminus \{-3; 3\}$$

$$\textcircled{a = ?}$$

$$\text{Za: } x^2 + a \neq 0 \quad (\Leftrightarrow)$$

$$\text{Za: } \uparrow \uparrow (x-3)(x+3) \neq 0$$

$$x^2 - 9 \neq 0$$

$$a = -9 \quad \text{spr.} \quad f(x) = \frac{5}{x^2 - 9}$$

$$\text{Za: } x^2 - 9 \neq 0$$

$$(x-3)(x+3) \neq 0$$

$$\underline{x+3} \wedge \underline{x \neq -3} \quad \mathbb{R} \setminus \{-3; 3\}$$

$$f(x) = \sqrt{2x+10a}$$

$$\textcircled{II}: x \in \langle 1; +\infty \rangle$$

od antan $\textcircled{D}$

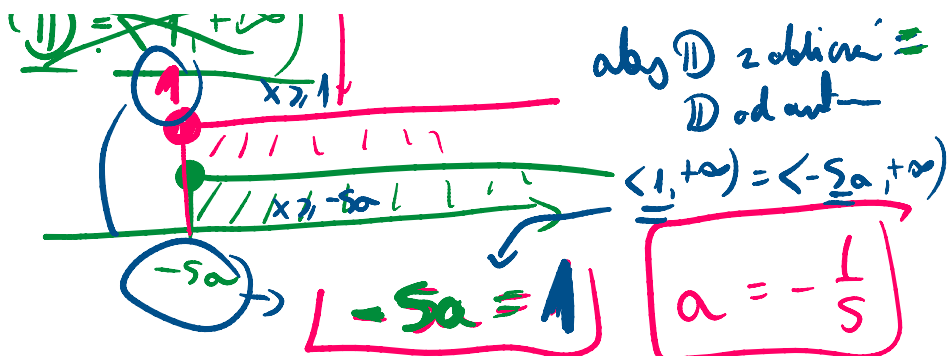
$$\textcircled{II} = \langle 1; +\infty \rangle$$

abs $\textcircled{II}$ 2 obliczenia =
1 punkt

↓
Zau: $2x + 10a \geq 0$

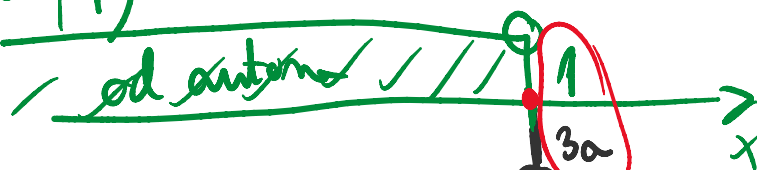
$x \geq -5a$

① z obliczenia.



$f(x) = \frac{1}{\sqrt{3a-x}}$

①: $(-\infty, 1)$



Zau: $3a - x > 0$

$3a > x$

$x < 3a$

$3a = 1$
 $a = \frac{1}{3}$

①:

$$e) g(x) = \sqrt{4-x^2} + \sqrt{x-2} + \sqrt{5}$$

$$a \cdot b \geq 0$$

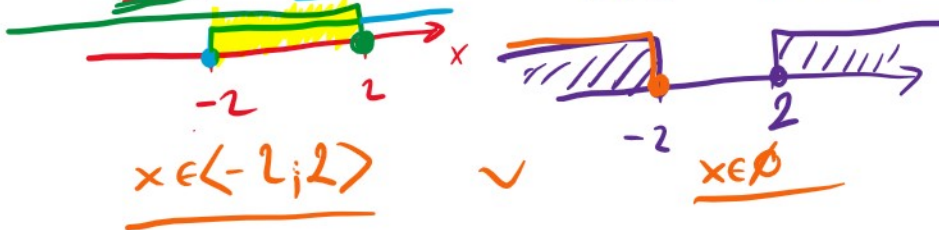
$$\frac{-2-5}{(-2) \cdot 4}$$

$$0 \cdot 3$$

$$\text{zak: } (2-x)(2+x) \geq 0 \wedge x-2 \geq 0$$

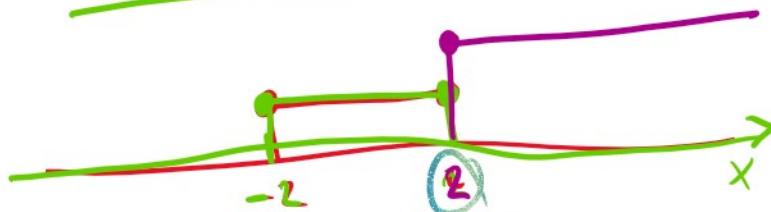
$$(2-x \geq 0 \wedge 2+x \geq 0) \vee (2-x \leq 0 \wedge 2+x \leq 0) \wedge x \geq 2$$

$$\begin{matrix} x \leq 2 & \wedge & x \geq -2 \\ x \geq 2 & \wedge & x \leq -2 \end{matrix}$$



$$x \in (-2, 2)$$

$$\wedge x \in (2, +\infty)$$



$$\textcircled{1}: x \in \{2\}$$

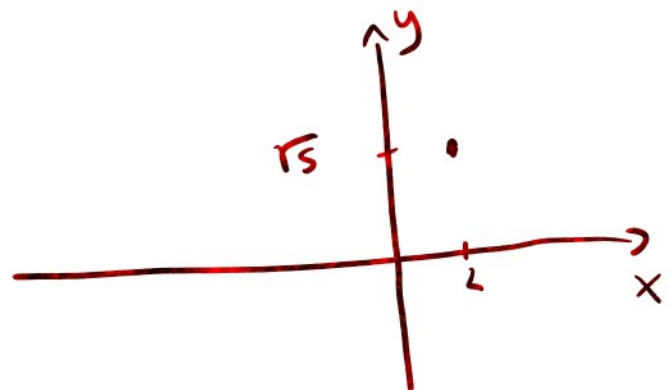
$$\text{ZW: } y \in$$

$$e) g(x) = \sqrt{4-x^2} + \sqrt{x-2} + \sqrt{5}$$

$$g(2) = \sqrt{4-4} + \sqrt{2-2} + \sqrt{5} = \sqrt{5} \quad (2; \sqrt{5})$$

$$\textcircled{1}: x \in \{2\}$$

$$\text{ZW: } y \in \{\sqrt{5}\}$$



$$4-x^2 \geq 0$$

$$x^2 \leq 4$$

$$|x| \leq 2$$

$$\sqrt{x^2} = |x|$$

$$|x| \leq 2 \iff -2 \leq x \leq 2$$

$$|x| \leq 2 \Leftrightarrow x \leq 2 \wedge x \geq -2$$

$$x \in (-2, 2)$$



$$n) f(x) = \sqrt{x \cdot \frac{|1+x| - |1-x|}{|1+x| + |1-x|}}, \quad x \in \mathbb{R}_+$$

$$x \geq 0$$

$$\textcircled{II}: \quad x \cdot \frac{|1+x| - |1-x|}{|1+x| + |1-x|} \geq 0$$

$$|1+x| - |1-x| \geq 0$$

$$\underbrace{|1+x|}_{\geq 0} \geq \underbrace{|1-x|}_{\geq 0} \quad |()|^2$$

$$(1+x)^2 \geq (1-x)^2$$

$$x + 2x + x^2 - (x - 2x + x^2) \geq 0$$

$$4x \geq 0$$

$$x \geq 0$$

$$\textcircled{II} : x \in \mathbb{R}_+$$

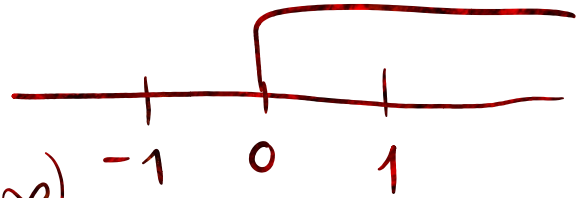
$$x \in (0, +\infty)$$

$$x \in (0, +\infty)$$

$$x \in \mathbb{R}_+ \cup \{0\}$$

$$\mathbb{R} = \mathbb{R}_- \cup \{0\} \cup \mathbb{R}_+$$

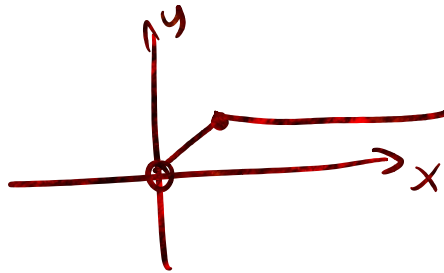
$$n) f(x) = \sqrt{x \cdot \frac{|1+x| - |1-x|}{|1+x| + |1-x|}}, \quad x \in \mathbb{R}_+$$



$$\begin{cases} x \in (0, 1) \\ f(x) = \sqrt{x \cdot \frac{1+x-1+x}{1+x+1-x}} \checkmark \\ = \sqrt{x^2} = \underline{|x|} \end{cases}$$

$$\begin{cases} x \in (1, +\infty) \\ f(x) = \sqrt{x \cdot \frac{1+x+1-x}{1+x-1+x}} = \sqrt{\frac{x}{x}} = 1 \end{cases}$$

$$\begin{aligned} f(x) &= |x| = x \\ x &\in (0, 1) \end{aligned}$$



2021-12-22

wtorek, 21 grudnia 2021 07:23