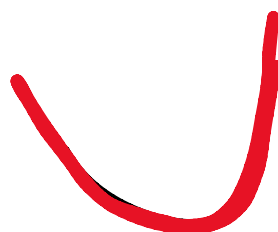
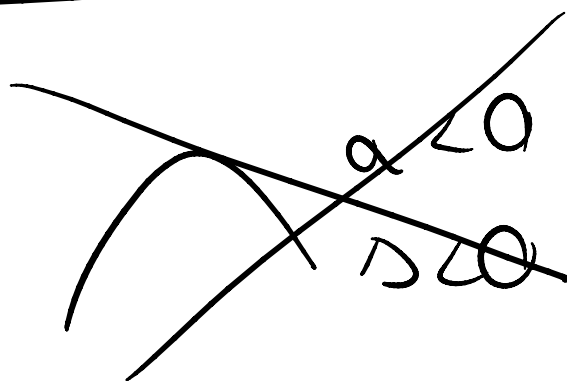


... $\rightarrow 0$
NAD
ROD

 $a \rightarrow a$
 $D < 0$

 $a < 0$
 $D < 0$

 $x \in \mathbb{R}$

$$2x^2 - 10x - 12 \leq 0$$

$$1, 2, 3, \dots, 25$$

$$\frac{10}{2 \cdot 5} = 10$$

$$\frac{S \cdot P}{A} \quad \text{dużo}$$

$$1 \cdot 5 \cdot 2 \cdot 5 \cdot 3 \cdot 5 \cdot 4 \cdot 5 \cdot 5 \cdot 5 = 5^6$$

$$25 \cdot 4 = 100$$

$$5 \cdot 5 \cdot 2 \cdot 2$$

$$000000$$

$$1) \quad 0 = \frac{S \cdot P}{A}$$

$$\underline{1 \cdot 2 \cdot 3 \cdot \dots \cdot 100}$$

$$5^3 = \underline{125}$$

$$100 : 5 = \boxed{20}$$

$$20 : 5 = \boxed{4}$$

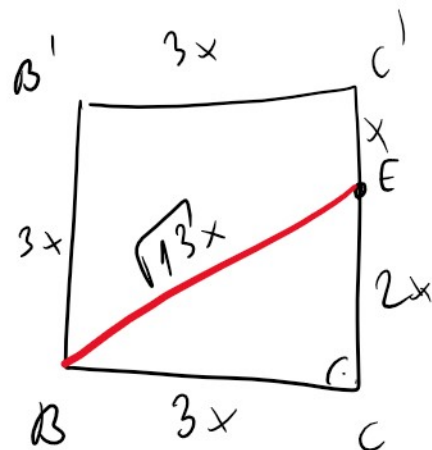
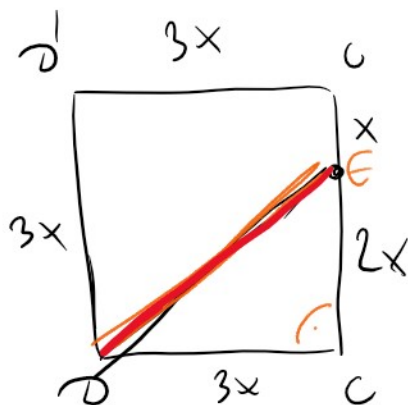
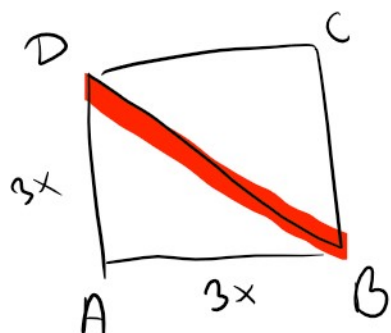
$$\underline{4 : 5 = 0} \text{ } \sim 4$$

$0, 1, 2, \dots, 9$ $\boxed{9}$ $h(0) =$

$$x = \underline{9 \cdot k} + 3 \quad \underline{k \in \mathbb{C}}$$

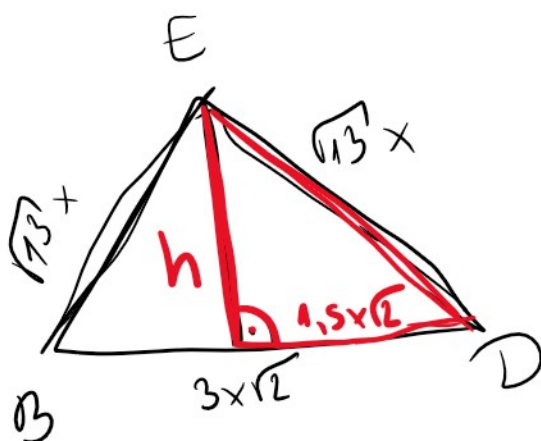
$$\boxed{7k+2}, \quad \underbrace{7 \cdot k + 2}_{\boxed{7k+9}} + 7, \quad \underbrace{7k+16}$$

$$\underbrace{5k+3}, \quad \underbrace{5k+8}, \quad \underbrace{5k+13}$$



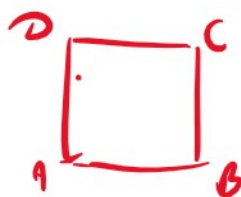
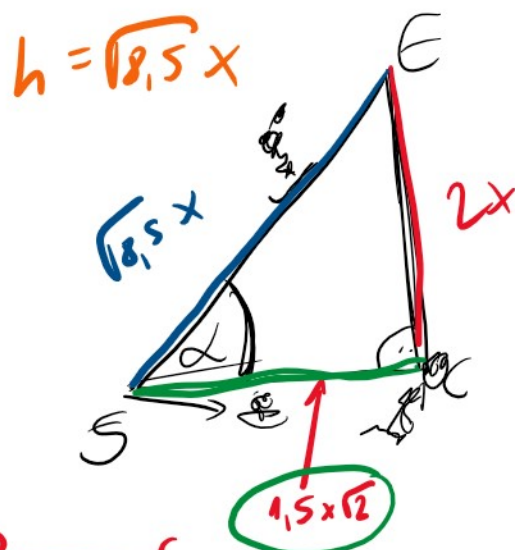
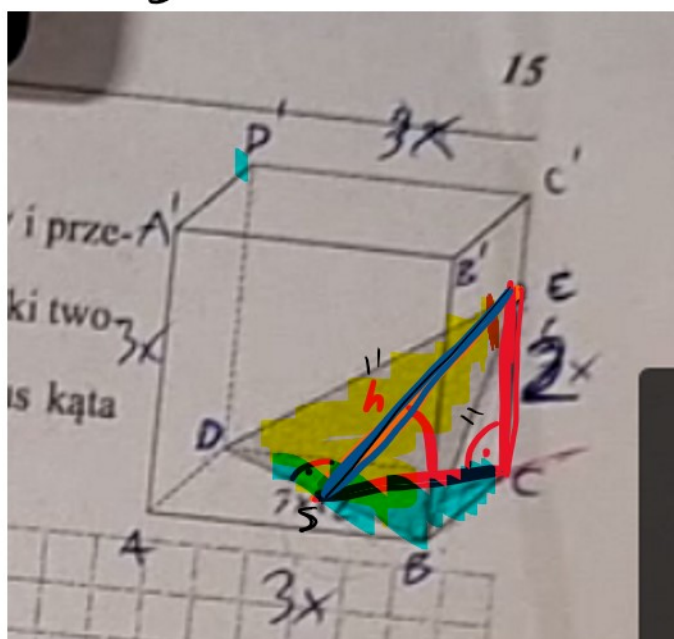
$$BE^2 = \sqrt{13x^2} = \sqrt{13}x$$

$$\sqrt{x^2} = |x| = x$$



$$2,25 \cdot 2 \cdot x^2 + h^2 = 13x^2$$

$$h^2 = (13 - 4,5)x^2$$



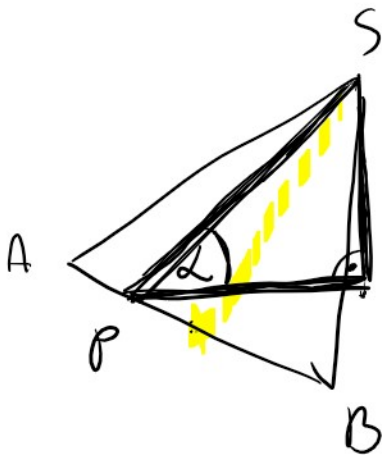
$$\cos(180-\alpha) = -\cos\alpha$$

$$\cos\alpha = \frac{1,5 \times \sqrt{2}}{\sqrt{8,5} \times} = \sqrt{\left(\frac{1,5 \cdot \sqrt{2}}{\sqrt{8,5}}\right)^2}$$

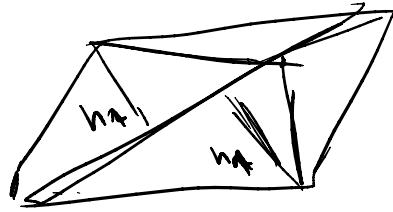


$$\frac{1}{2} = \sqrt{\left(\frac{1}{2}\right)^2} \quad \parallel \quad \frac{\sqrt{2,25 \cdot 2}}{8,5} = \frac{4,5 \cdot \sqrt{2}}{8,5 \cdot 2} \sqrt{\frac{9}{17}} = \frac{3}{\sqrt{17}}$$

$$\cos\alpha = \frac{3}{\sqrt{17}} = \frac{3\sqrt{17}}{17}$$

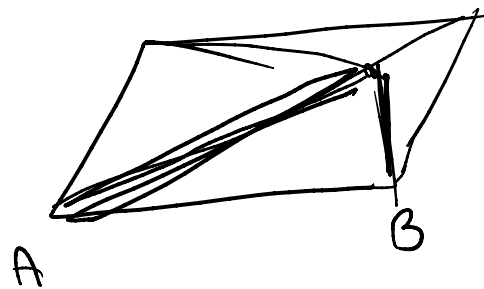


Z:



T: $P_{ABP} = P_{ADP}$

D: $\triangle ABC \equiv \triangle ADC$
 $\Downarrow$
 $h_1 = h_2 = h$



$$P_{ABP} = \frac{AP \cdot h}{2}$$

$$P_{ADP} = \frac{AP \cdot h}{2}$$

$$\Rightarrow P_{ABP} = P_{ADP} \quad \text{c.m.d.}$$

Zadanie 27. (0-2)

Wykaż, że prawdziwa jest nierówność $4^{24} > 3^{30}$.

$$\begin{aligned}
 4^{24} &= (4^4)^6 > (3^5)^6 \\
 (64 \cdot 4)^6 &> (81 \cdot 3)^6 \\
 (256)^6 &> (243)^6 \\
 256 &> 243
 \end{aligned}$$

$$\left(\frac{1}{2}\right)^5 > \left(\frac{1}{2}\right)^{10} \quad (5 < 10)$$

$$a^m > a^m$$

$$a > 1$$

$$m > m$$

$$a \in (0, 1)$$

$$m < m$$

$$2^0 = 1$$

$$2^{-1} = \frac{1}{2}$$

$$y = 2^x$$

$$y = \left(\frac{1}{2}\right)^x$$

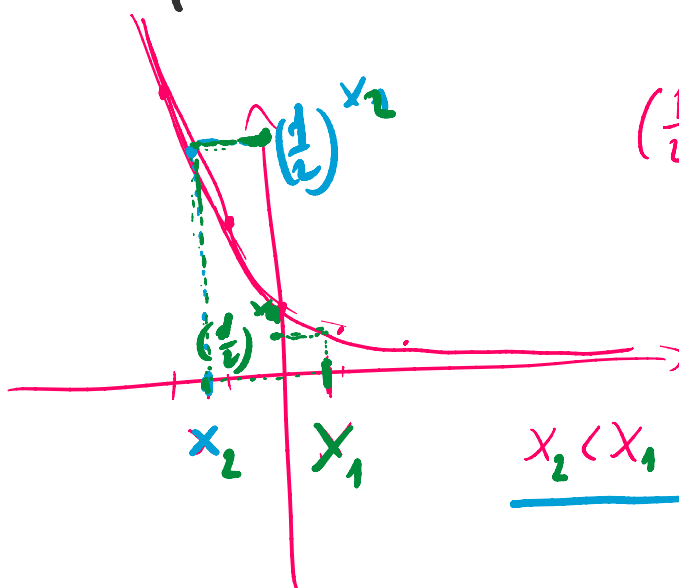
$$f \nearrow$$

$$2^{x_1} > 2^{x_2} \Leftrightarrow x_1 > x_2$$

$$\left(\frac{1}{2}\right)^{-2} = \left(\frac{2}{1}\right)^2 = 2^2 = 4$$

$$y = \left(\frac{1}{2}\right)^x$$

$$a^0 = 1$$



$$f \searrow$$

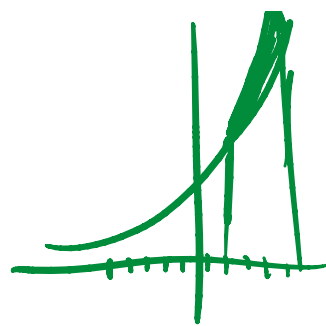
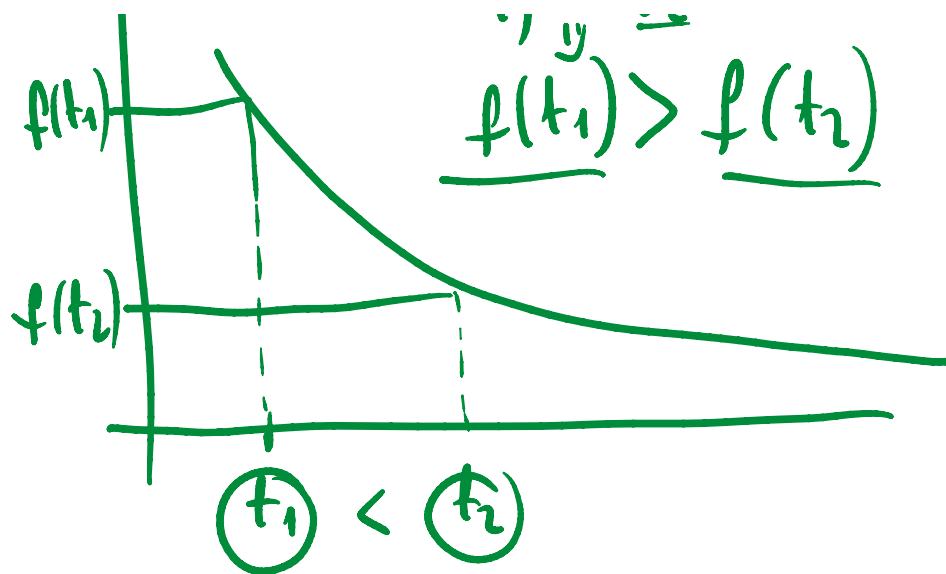
$$x_2 < x_1 \Rightarrow \left(\frac{1}{2}\right)^{x_2} > \left(\frac{1}{2}\right)^{x_1}$$

$$m \perp$$

$$t_1 < t_2$$

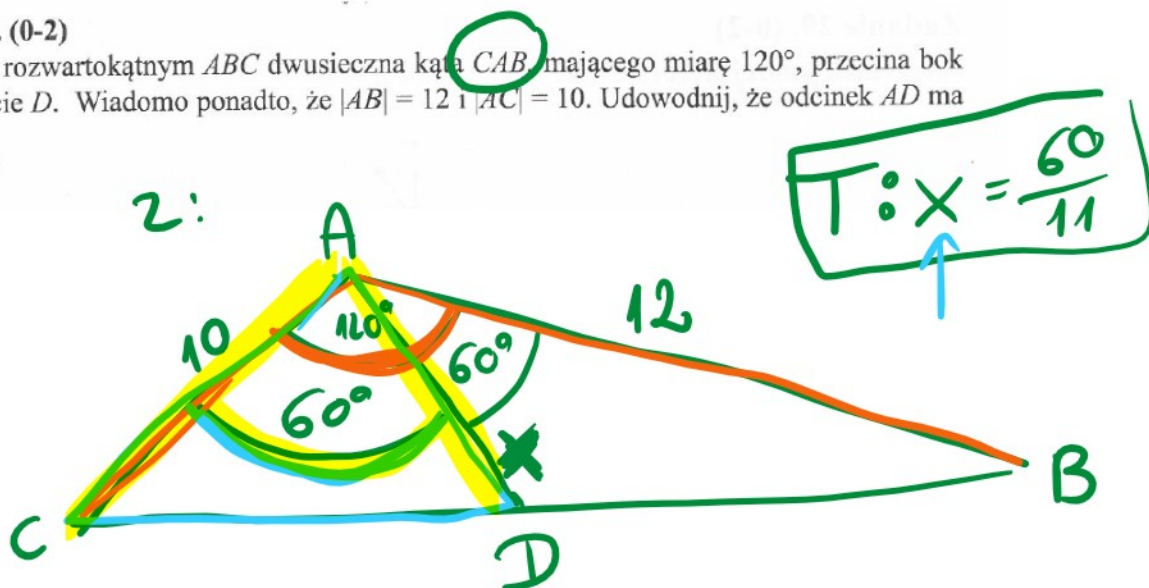
$$n(1, 1) \setminus p(1, 1)$$

$$1 \perp$$



Zadanie 28. (0-2)

W trójkącie rozwartokątnym ABC dwusieczna kąta CAB mającego miarę 120° , przecina bok BC w punkcie D . Wiadomo ponadto, że $|AB| = 12$ i $|AC| = 10$. Udowodnij, że odcinek AD ma długość $\frac{60}{11}$.



①; TAB STR 8 $P_{\Delta ABC} = \frac{1}{2} 10 \cdot 12 \sin 120^\circ =$

- Wzory na pole trójkąta

$$P_{\Delta ABC} = \frac{1}{2} \cdot a \cdot h_a = \frac{1}{2} \cdot b \cdot h_b = \frac{1}{2} \cdot c \cdot h_c$$

$$P_{\Delta ABC} = \frac{1}{2} a \cdot b \cdot \sin \gamma = \frac{1}{2} a \cdot c \cdot \sin \beta = \frac{1}{2} b \cdot c \cdot \sin \alpha$$

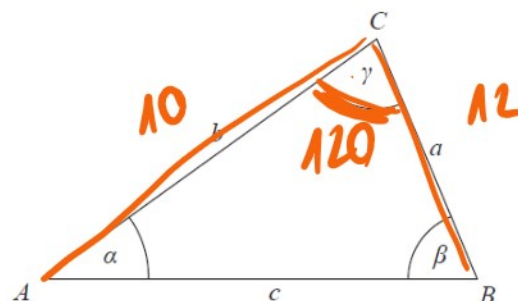
$$P_{\Delta ABC} = \frac{1}{2} a^2 \frac{\sin \beta \cdot \sin \gamma}{\sin \alpha} = \frac{1}{2} b^2 \frac{\sin \alpha \cdot \sin \gamma}{\sin \beta} = \frac{1}{2} c^2 \frac{\sin \alpha \cdot \sin \beta}{\sin \gamma}$$

$$P_{\Delta ABC} = \frac{abc}{4R}$$

$$P_{\Delta ABC} = 2R^2 \cdot \sin \alpha \cdot \sin \beta \cdot \sin \gamma$$

$$P_{\Delta ABC} = rp$$

$$P_{\Delta ABC} = \sqrt{p(p-a)(p-b)(p-c)}$$



- Wybrane wzory redukcyjne

$$\sin(90^\circ - \alpha) = \cos \alpha$$

$$\cos(90^\circ - \alpha) = \sin \alpha$$

$$\sin(90^\circ + \alpha) = \cos \alpha$$

$$\cos(90^\circ + \alpha) = -\sin \alpha$$

$$\checkmark \sin(180^\circ - \alpha) = \sin \alpha$$

$$\cos(180^\circ - \alpha) = -\cos \alpha$$

$$\operatorname{tg}(180^\circ - \alpha) = -\operatorname{tg} \alpha$$

$$\sin(180^\circ + \alpha) = -\sin \alpha$$

$$\cos(180^\circ + \alpha) = -\cos \alpha$$

$$\operatorname{tg}(180^\circ + \alpha) = \operatorname{tg} \alpha$$

$$\sin(120^\circ) = \sin(180^\circ - 60^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin(180^\circ - \alpha) = \sin \alpha$$

$$\sin(180^\circ - \alpha) = \sin \alpha$$

α	0°	30°	45°	60°	90°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\operatorname{tg} \alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	nie istnieje

$$P_{\triangle ABC} = 60 \cdot \frac{\sqrt{3}}{2} = 30\sqrt{3}$$

$$11 \times \frac{\sqrt{3}}{2} = 60 \frac{\sqrt{3}}{2}$$

$$x = \frac{60}{11}$$

cmd

9
20

$$Obv = 2\pi r$$

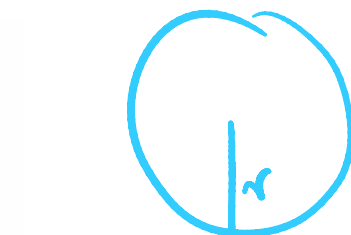
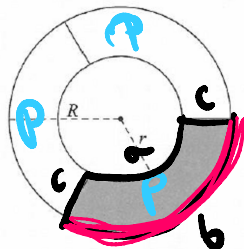
Zadanie 32. (0-4)

W 2017 roku miasto Racibórz obchodziło 800 lecie nadania praw miejskich. W związku z tym jubileuszem zorganizowano wiele różnych projektów. Jednym z nich był projekt *4 biegi*. Za każdy ukończony bieg można było otrzymać jedną z czterech części okolicznościowego medalu. Po złożeniu wszystkich części powstawał medal w kształcie koła (patrz rysunek).

$$C=4$$

$$a = \frac{1}{3} \cdot 20 \cdot \pi$$

$$b = \frac{1}{3} \cdot 28 \pi$$



$$P_k = \pi r^2 \text{ cm}^2$$

$$P_p = \pi R^2 - \pi r^2$$

Kawałki medalu stanowiące pierścień są identyczne. Oblicz pole i obwód zaznaczonej części medalu, przyjmując, że $r = 10$ cm i $R = 14$ cm.

$$P_p = P_{kd} - P_{kn} = \pi R^2 - \pi r^2 = \pi (R^2 - r^2) =$$

$$\pi \cdot (14^2 - 10^2) = \pi (196 - 100) = \underline{\underline{\pi \cdot 96}}$$

$$P_n = \frac{1}{3} P_p = 32\pi \text{ cm}^2$$

$$Obv = a + b + 2c = \frac{1}{3} \cdot 20\pi + \frac{1}{3} \cdot 28\pi + 8 = \frac{1}{3} \cdot 48\pi + 8 = 16\pi + 8 \text{ cm}$$

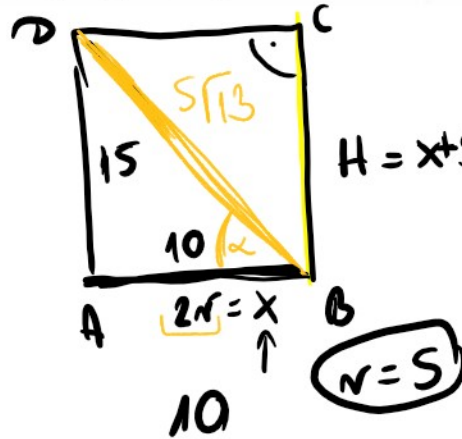
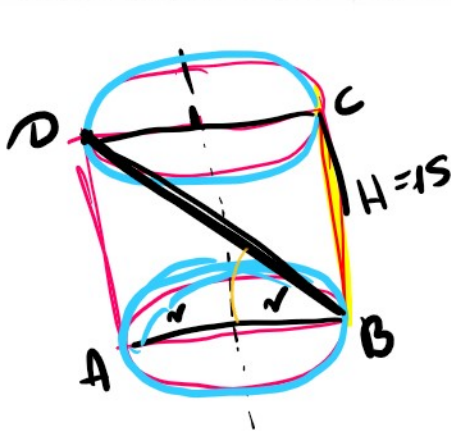
$$\frac{1}{3} (20\pi + 28\pi)$$

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3} = \frac{1}{3} (1+1)$$

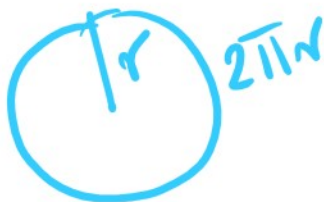
$$\begin{aligned} x - 13x &= -12x \\ 25 - 13 \cdot 25 &= -12 \cdot 25 \end{aligned}$$

Zadanie 34. (0-5)

Wysokość walca jest o 5 dłuższa od średnicy jego podstawy. Przekątna przekroju osiowego tego walca ma długość $5\sqrt{13}$. Oblicz pole powierzchni bocznej walca oraz miarę kąta nachylenia przekątnej przekroju osiowego do płaszczyzny podstawy.



$$P_b = 15 \cdot 2\pi \cdot 5 = 150\pi$$



$$x > 0$$

$$x^2 + 5x - 150 = 0$$

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-5 - 15}{2} = -10$$

$$x_2 = \frac{-5 + 25}{2} = 10$$

$$\Delta = b^2 - 4ac$$

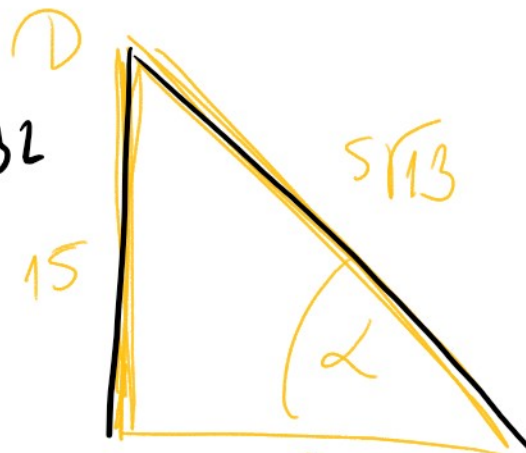
$$\Delta = 25 + 4 \cdot 150$$

$$\Delta = 5^2 - 4 \cdot (-150)$$

$$\Delta = 25$$

$$\sin \alpha = \frac{15}{5\sqrt{13}} = \frac{3}{\sqrt{13}} \approx 0,832$$

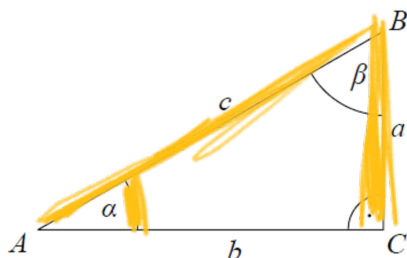
$$\frac{3}{\sqrt{13}} \approx$$



13 ~



- Definicje funkcji trygonometrycznych kąta ostrego w trójkącie prostokątnym



$$\sin \alpha = \frac{a}{c}$$

$$\sin \beta = \frac{b}{c}$$

$$\cos \alpha = \frac{b}{c}$$

$$\cos \beta = \frac{a}{c}$$

$$\operatorname{tg} \alpha = \frac{a}{b}$$

$$\operatorname{tg} \beta = \frac{b}{a}$$

2021-12-16

czwartek, 16 grudnia 2021

19:59

$\alpha [^\circ]$	$\sin \alpha$ $\cos \beta$	$\operatorname{tg} \alpha$	$\beta [^\circ]$
0	0,0000	0,0000	90
1	0,0175	0,0175	89
2	0,0349	0,0349	88
3	0,0523	0,0524	87
4	0,0698	0,0699	86
5	0,0872	0,0875	85
6	0,1045	0,1051	84
7	0,1219	0,1228	83
8	0,1392	0,1405	82
9	0,1564	0,1584	81
10	0,1736	0,1763	80
11	0,1908	0,1944	79
12	0,2079	0,2126	78
13	0,2250	0,2309	77
14	0,2419	0,2493	76
15	0,2588	0,2679	75
16	0,2756	0,2867	74
17	0,2924	0,3057	73
18	0,3090	0,3249	72
19	0,3256	0,3443	71
20	0,3420	0,3640	70
21	0,3584	0,3839	69
22	0,3746	0,4040	68
23	0,3907	0,4245	67
24	0,4067	0,4452	66
25	0,4226	0,4663	65
26	0,4384	0,4877	64
27	0,4540	0,5095	63
28	0,4695	0,5317	62
29	0,4848	0,5543	61
30	0,5000	0,5774	60
31	0,5150	0,6009	59
32	0,5299	0,6249	58
33	0,5446	0,6494	57
34	0,5592	0,6745	56
35	0,5736	0,7002	55
36	0,5878	0,7265	54
37	0,6018	0,7536	53
38	0,6157	0,7813	52
39	0,6293	0,8098	51
40	0,6428	0,8391	50
41	0,6561	0,8693	49
42	0,6691	0,9004	48
43	0,6820	0,9325	47
44	0,6947	0,9657	46
45	0,7071	1,0000	45

$\alpha [^\circ]$	$\sin \alpha$ $\cos \beta$	$\operatorname{tg} \alpha$	$\beta [^\circ]$
46	0,7193	1,0355	44
47	0,7314	1,0724	43
48	0,7431	1,1106	42
49	0,7547	1,1504	41
50	0,7660	1,1918	40
51	0,7771	1,2349	39
52	0,7880	1,2799	38
53	0,7986	1,3270	37
54	0,8090	1,3764	36
55	0,8192	1,4281	35
56	0,8290	1,4826	34
57	0,8387	1,5399	33
58	0,8480	1,6003	32
59	0,8572	1,6643	31
60	0,8660	1,7321	30
61	0,8746	1,8040	29
62	0,8829	1,8807	28
63	0,8910	1,9626	27
64	0,8988	2,0503	26
65	0,9063	2,1445	25
66	0,9135	2,2460	24
67	0,9205	2,3559	23
68	0,9272	2,4751	22
69	0,9336	2,6051	21
70	0,9397	2,7475	20
71	0,9455	2,9042	19
72	0,9511	3,0777	18
73	0,9563	3,2709	17
74	0,9613	3,4874	16
75	0,9659	3,7321	15
76	0,9703	4,0108	14
77	0,9744	4,3315	13
78	0,9781	4,7046	12
79	0,9816	5,1446	11
80	0,9848	5,6713	10
81	0,9877	6,3138	9
82	0,9903	7,1154	8
83	0,9925	8,1443	7
84	0,9945	9,5144	6
85	0,9962	11,4301	5
86	0,9976	14,3007	4
87	0,9986	19,0811	3
88	0,9994	28,6363	2
89	0,9998	57,2900	1
90	1,0000	—	0

0,832

 $\alpha \approx 565^\circ$

1 2 3 4 5 6 7 8 9

$$N = \{1, 3, 5, 7, 9\}$$

$$P = \{0, 2, 4, 6, 8\}$$

Zadanie 31. (0-2)

Uzasadnij, że liczb naturalnych czterocyfrowych, w których co najmniej trzy cyfry są parzyste, jest 2625.

✗

$$\begin{array}{l} \textcircled{N} \text{ } \underline{P} \text{ } \underline{P} \text{ } \underline{P} \times 1 \\ \textcircled{P} \text{ } \underline{N} \text{ } \textcircled{P} \text{ } \underline{P} \times 3 \end{array}$$

$$5^4 + 5^3 \cdot 12 = 5^3 (5 + 12)$$

$$\underline{125 \cdot 17}$$

$$N P P P$$

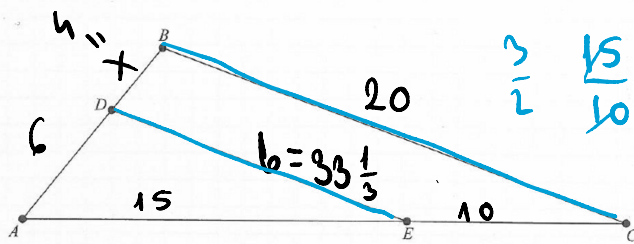
$$5 5 5 5 = 5^4$$

$$P N P P \times 3$$

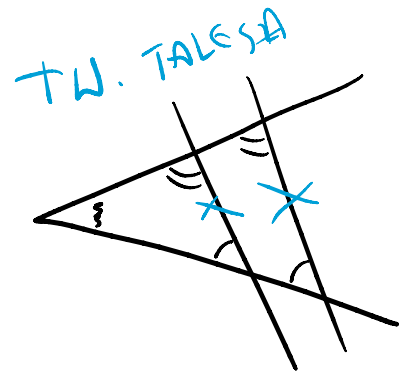
$$4 \cdot 5 \cdot 5 \cdot 5 \cdot 3 = 5^3 \cdot 12$$

Zadanie 27. (0-2)

Trójkąt ABC podzielono odcinkiem ED równoległym do boku BC (patrz rysunek poniżej).



$$\frac{3}{2} \cdot \frac{15}{10} = \frac{6}{x} \quad (4)$$



Przyjmując, że $|AD| = 6$, $|AE| = 15$, $|EC| = 10$ i $|BC| = 20$, oblicz obwód czworokąta $BCED$.

$$\text{Obw} = 10 + 20 + x + b$$

$$\frac{10 + 20 + x + b}{30 + x + b}$$

$$\triangle AED \sim \triangle ACB \Rightarrow \frac{6}{5} = \frac{15}{25} = \frac{6}{6+x} = \frac{20}{b}$$

$$\frac{6}{6+x} = \frac{3}{5} \Rightarrow$$

$$x = 4$$

$$6 + x = \frac{2 \cdot 5}{1} \quad | -6$$

$$x = 10 - 6 = 4$$

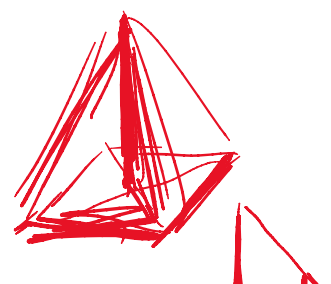
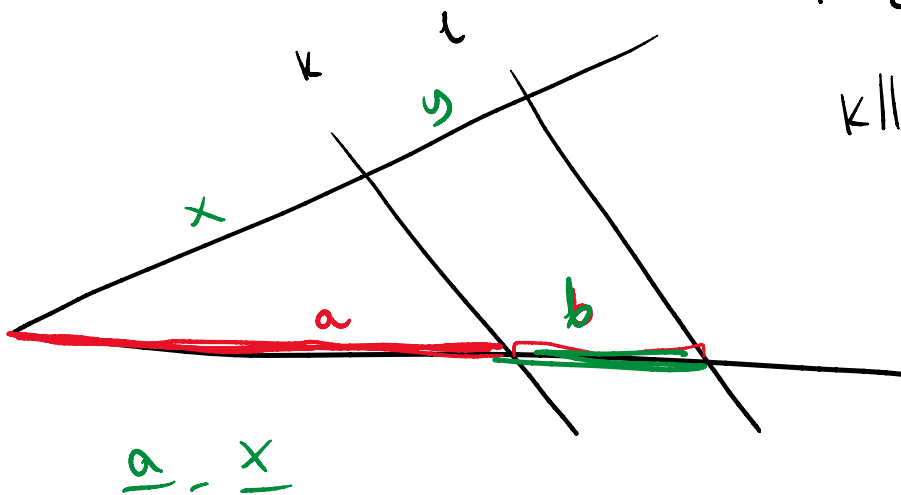
$$\frac{20}{b} = \frac{3}{5}$$

$$b = \frac{20 \cdot 5}{3} = \frac{100}{3} = 33 \frac{1}{3}$$

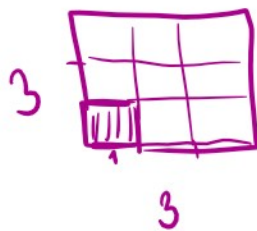
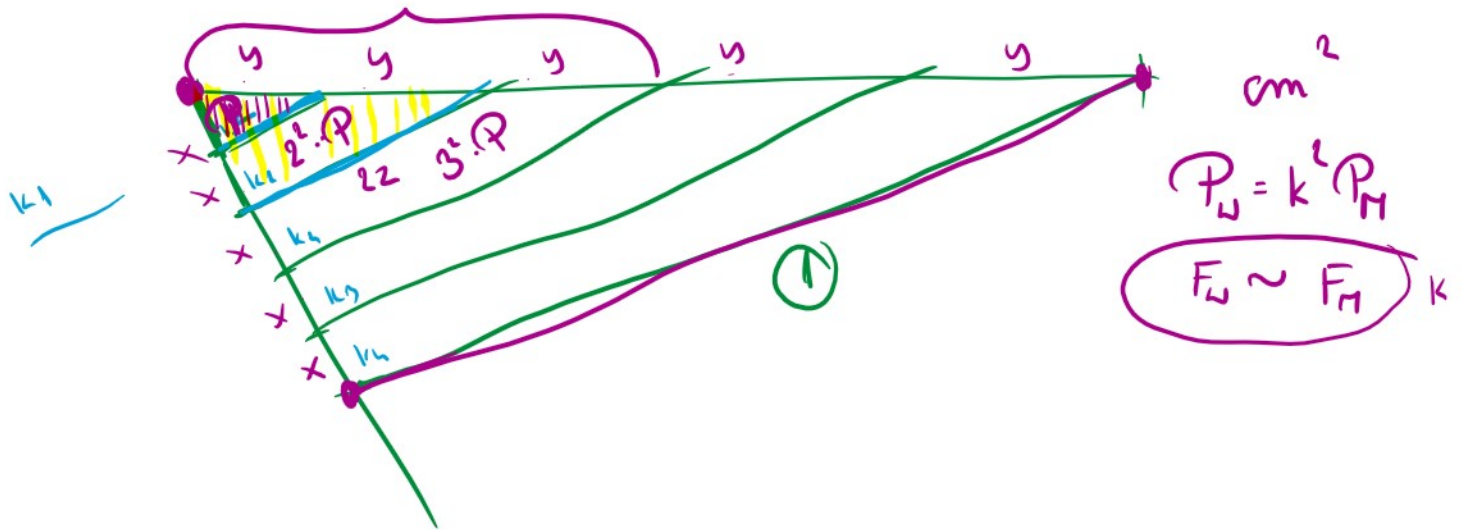
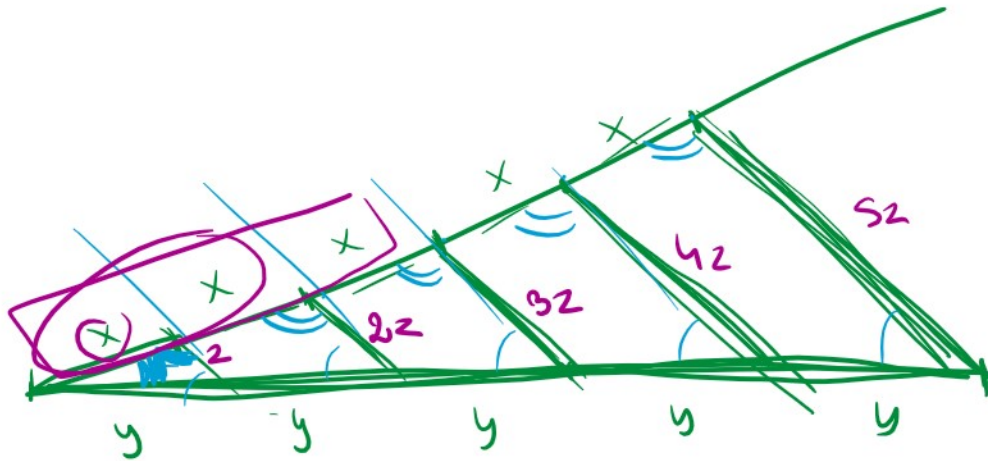
odp $67 \frac{1}{3}$

TALES

$k \parallel l$



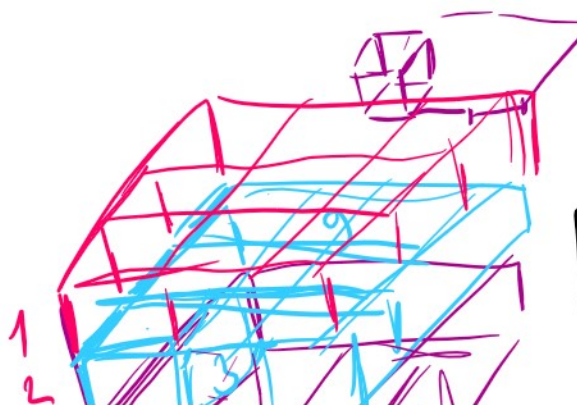
$$\frac{a}{b} = \frac{x}{y}$$



$$k=3$$

$$P_d = 3 \cdot m \cdot 3 \cdot m = 9m^2 = \underline{9P_m}$$

3 · 3
3 · 3

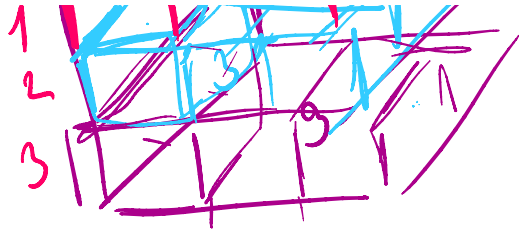


$$F_m \sim F_d \quad k$$

$$V_m \cdot k^3 = V_d$$

$$3 \cdot 3$$

$$(3 \cdot 3) \cdot 3$$

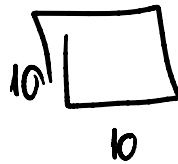


$$k^3$$

$$(9 \cdot 3) \quad 3^3 = 27 \text{ kostet hoch}$$

$$P_m \cdot k^2 = P_d$$

$$1 \text{ dm}^2 = 100 \text{ cm}^2$$



$$1 \text{ dm} = 10 \text{ cm}$$

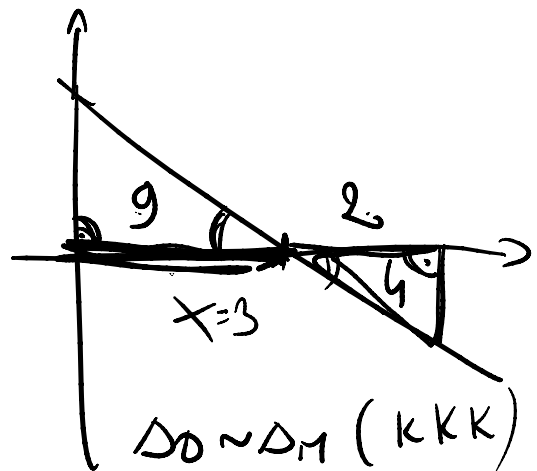
$$1 \text{ dm}^2 = 10^2 \text{ cm}^2$$

$$1 \text{ dm}^3 = 1000 \text{ cm}^3$$

$$S \cdot S \cdot S = S^3$$

hier
sich v. 1. nach

$$P \rightarrow \frac{\text{cm}^2}{k^2}$$



$$\Delta O \sim \Delta M (KKK) \quad (x=3)$$

$$\left(\frac{9}{4}\right) = \left(k^2\right) \Rightarrow \frac{9}{4} = k^2 = \frac{9}{2} = \frac{x}{2}$$

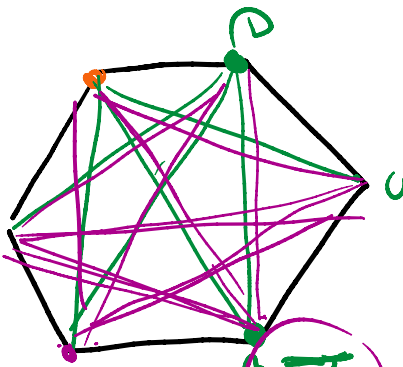
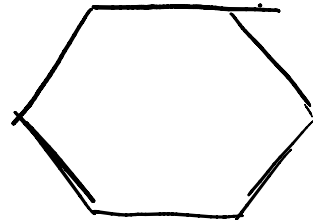
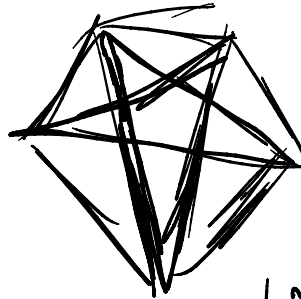
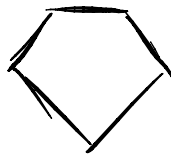
Zadanie 28. (0-2)

Udowodnij, że jedynym wielokątem, mającym tyle samo boków co przekątnych, jest pięciokąt.

$$Z: l_b = l_p$$

$$T: n = 5$$

①:



$n - l_{b,p}$

n

6:

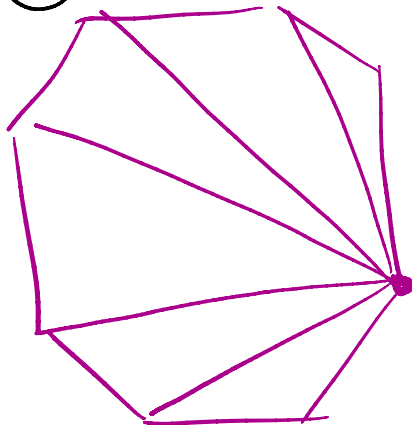
$$\frac{3 \cdot 6}{2} =$$

n :

$$\frac{n \cdot (n-3)}{2}$$

$$\overrightarrow{BD} = \overrightarrow{DB}$$

$$\frac{8 \cdot 5}{2} = 20$$



$$\frac{20 \cdot 17}{2} =$$

8: 20: 17: 3, 4, ..., 19

13, 14, 15, ..., 19

$$\frac{n(n-3)}{2} - \text{linia przek}$$

$$\frac{5 \cdot (5-3)}{2} = 5$$

FUJ

n - linbo //

$$n = \frac{n(n-3)}{2}$$

$$n \in \mathbb{N}_+ \\ n > 2$$

$$2n = n(n-3)$$

$$2n = n^2 - 3n$$

$$5n = n^2 \xrightarrow{\text{r.l.v.}} -n^2 + 5n = 0$$

$$\boxed{n=5} \vee n=0$$

$$\text{jed } n = \textcircled{n=5}$$

$$-n^2 + 5n = 0$$

$$-n(n-5) = 0$$

$$-n=0 \vee n-5=0$$

$$\textcircled{n=0} \vee \textcircled{n=5}$$

$$\boxed{n=5}$$

$$\boxed{n=7}$$

$$\boxed{\frac{0}{a}=0} \quad \text{FVJ}$$

$$\boxed{0 \cdot a = 0}$$

$$\boxed{-4 \cdot 0 = 0}$$

$$\underline{ax^2 + bx + c = 0}$$

$$a = -1 \quad \Delta = 25 - 4 \cdot (-1) \cdot 0 = 25 \\ b = 5 \quad \sqrt{\Delta} = 5 \\ c = 0$$

$$n_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-5 - 5}{-2} = \frac{-10}{-2} = 5$$

$$n_2 = \frac{-b + \sqrt{\Delta}}{-2} = 0$$

$$\begin{matrix} a \\ b \\ c=0 \end{matrix} \Rightarrow \text{vytaz pd} \\ \text{mens } \underline{ax}$$

$$ax^2 + bx = 0$$

$$\underline{ax(x + \frac{b}{a}) = 0}$$

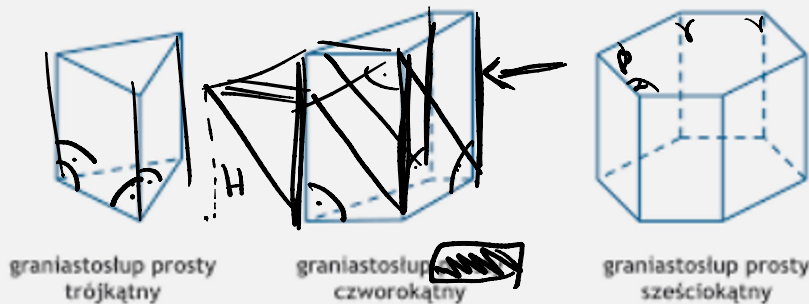
Zadanie 34. (0-4)

Pole powierzchni bocznej graniastosłupa prawidłowego czworokątnego, którego wysokość ma długość 4, jest równe 48. Wyznacz pole przekroju płaszczyzną przechodzącą przez przekątne

PROSTY



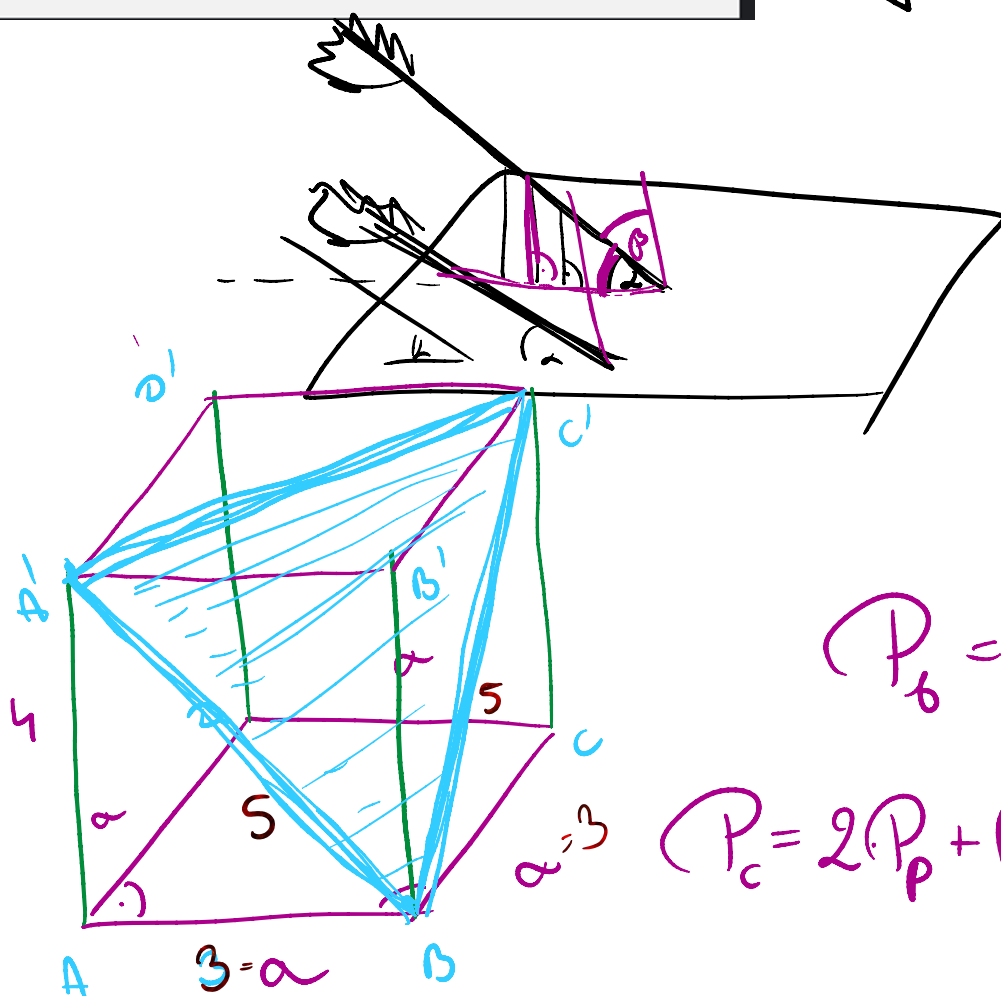
KUB



graniastosłup prosty trójkątny

graniastosłup prosty czworokątny

graniastosłup prosty sześciokątny



$$P_b = 48 = 4 \cdot 4 \cdot a$$

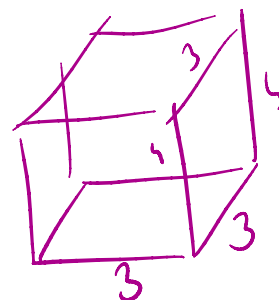
$$P_c = 2P_p + P_b$$

Zadanie 34. (0-4)

Pole powierzchni bocznej graniastosłupa prawidłowego czworokątnego, którego wysokość ma długość 4, jest równe 48. Wyznacz pole przekroju płaszczyzną przechodzącą przez przekątne ścian bocznych wychodzących z jednego wierzchołka.

$$48 = 4 \cdot 4 \cdot a \quad | :16$$

$$a = 3$$

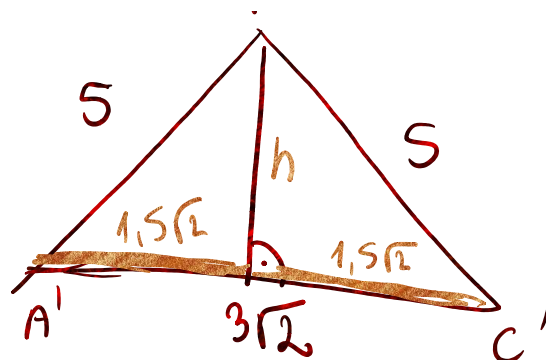
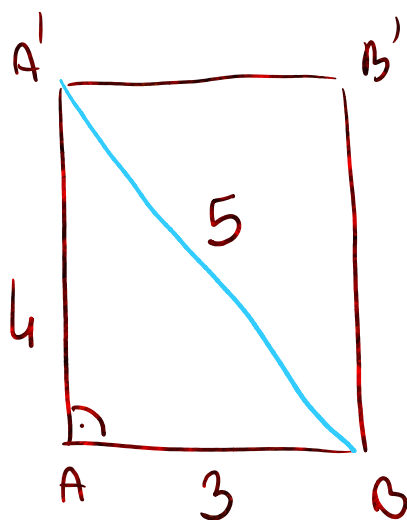


n

n

n



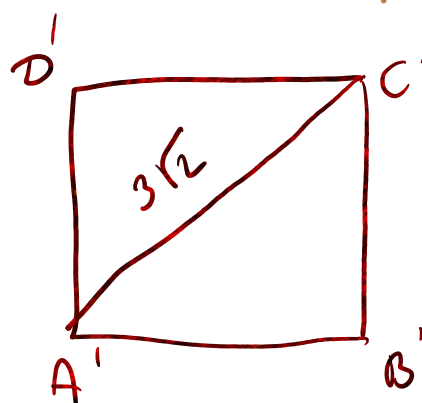


$$25 = \frac{9}{4} \cdot 2 + h^2$$

$$25 - 4,5 = h^2$$

$$\sqrt{20,5} = h$$

$$P = \frac{1}{2} \cdot 3\sqrt{2} \cdot \sqrt{20,5} = \frac{3}{2} \sqrt{41}$$



GRANIASTOSEK ;

GRANIASTOSEK. PROSTY ;

GR. PRAWIDŁOWY. :

ostros.

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2021-12-23

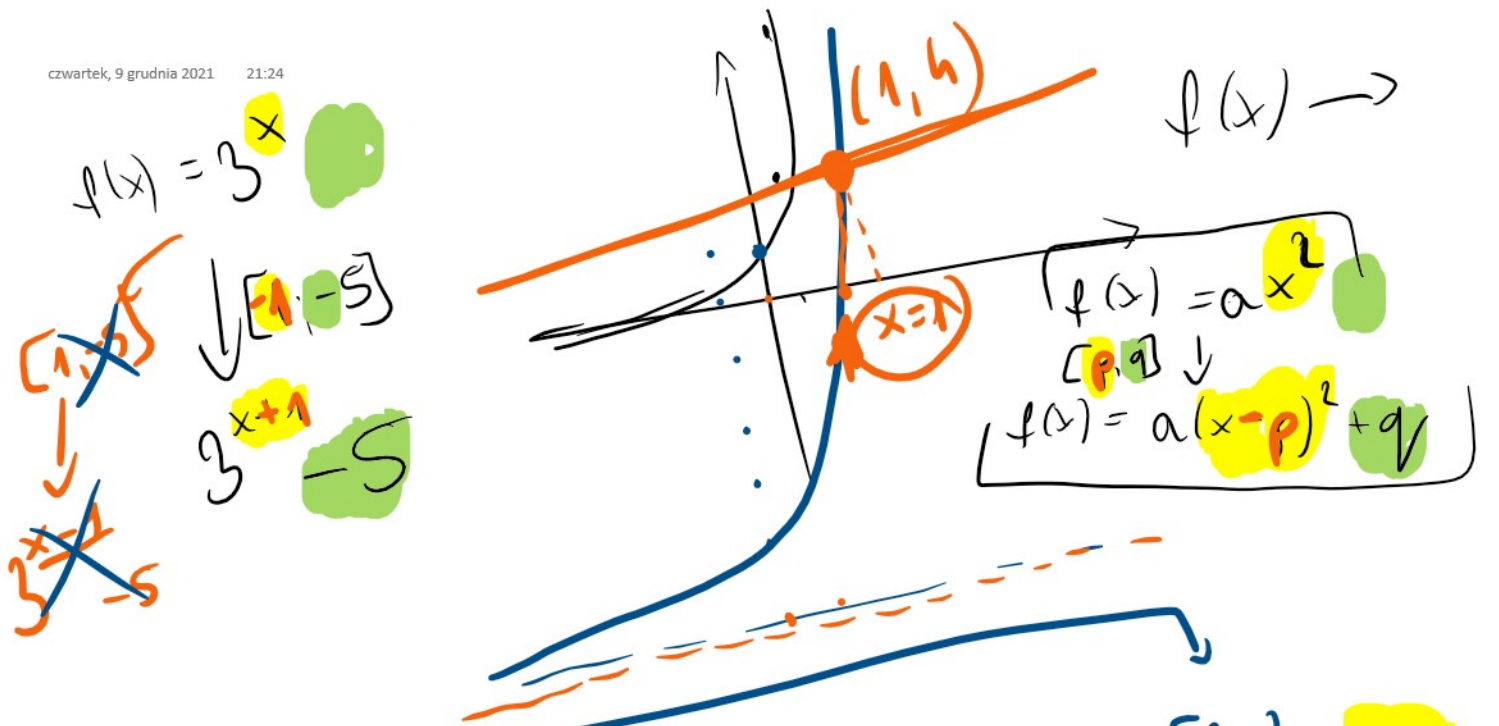
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2021-12-23

czwartek, 16 grudnia 2021 19:59

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$$1) f(x) \xrightarrow{[p, 0]} f(x-p)$$

$$2) f(x) \xrightarrow{[0, q]} f(x)+q$$

$$3^x \xrightarrow{[2, 0]} 3^{x-2}$$

$$3^x \xrightarrow{[0, -2]} 3^x - 2$$

$$3^x \xrightarrow{[0, 4]} 3^x + 4$$

$$3^x \xrightarrow{[-4, 0]} 3^{x+4}$$

$$3^x \xrightarrow{[-1, -5]} 3^{x+1} - 5$$

$$3^{x+1} - 5 = 4$$

$$3^{x+1} = 3^2$$

$$\left(\frac{1}{2}\right)^x \xrightarrow{[2,3)} \left(\frac{1}{2}\right)^{x-2} - 3$$