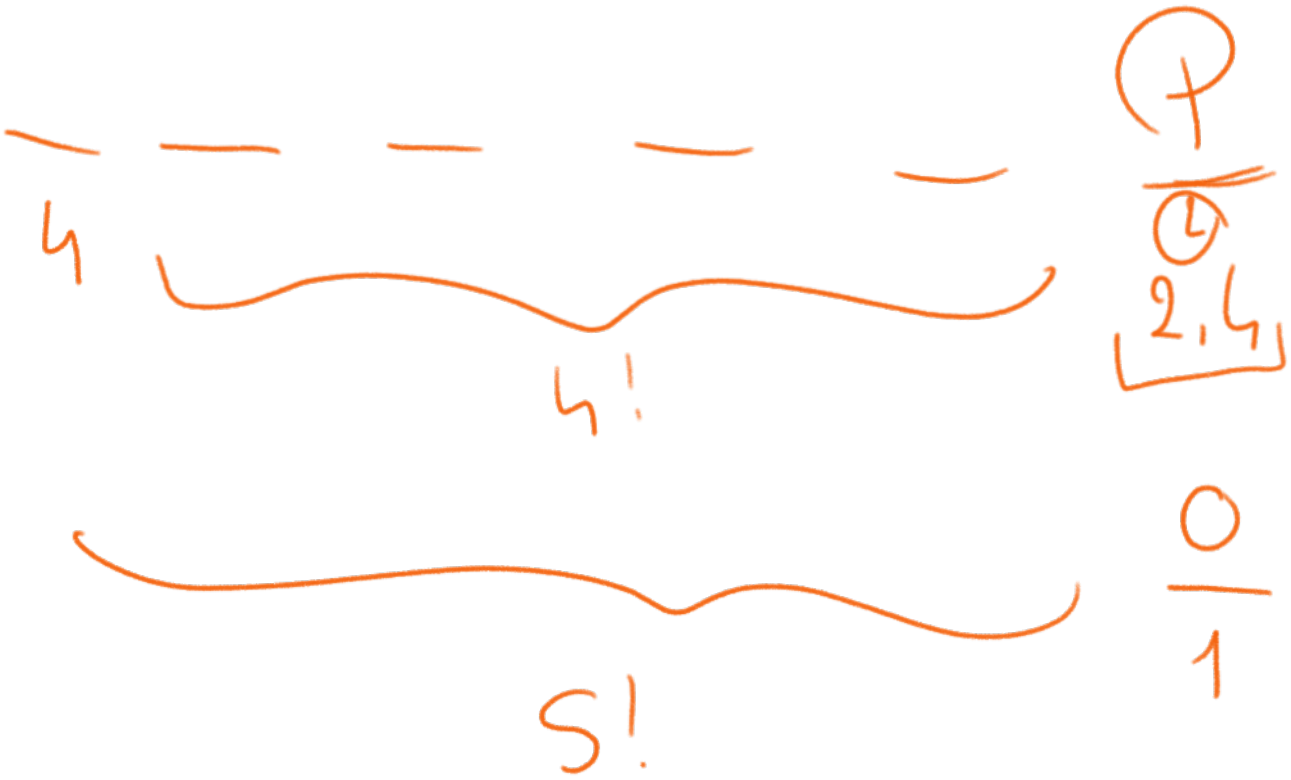
wtorek, 14 grudnia 2021 12:20



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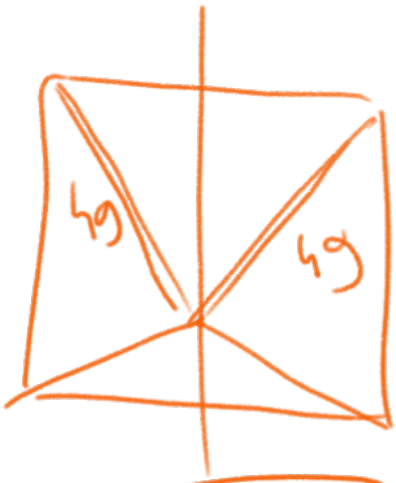

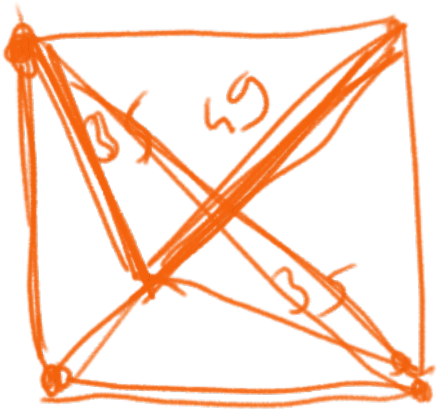

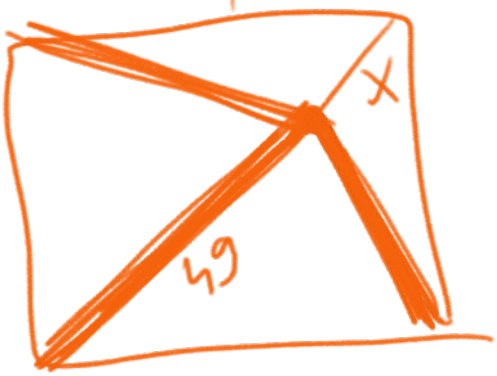

A

4. 2023 I/18

$$\sqrt{\frac{a^2+b^2+c^2}{3}} \geq \frac{a+b+c}{3}$$

$$\left(\frac{a^2+b^2+c^2}{3}\right)^{\frac{1}{2}} \geq \frac{a+b+c}{3}$$

$$\frac{a+b+c}{3} > 0$$

$$a+b+c > 0$$

$$\frac{a^2+b^2+c^2}{3} \geq \frac{(a+b+c)^2}{9}$$

$$\frac{3a^2+3b^2+3c^2-(a+b+c)^2}{9} \geq 0$$

$$2a^2+2b^2+2c^2-2ab-2ac-2bc \geq 0$$

$$2(a^2+b^2+c^2-ab-bc-ac) \geq 0$$

$$2(a+b+c)^2 - 6ab-6bc-6ac \geq 0$$

$$2a^2+2b^2+2b^2-2ab-2ac-2bc \geq 0$$

$$(a-b)^2+a^2+b^2-2ac+2c^2-2bc \geq 0$$

$$(a-b)^2+(a-c)^2+(b-c)^2 \geq 0$$

$\geq 0 \quad \geq 0 \quad \geq 0$

$$(a+b+c)(a+b+c) =$$

$$= a^2+ab+ac+ab+b^2+bc+ac+bc+c^2$$

$$a^2+2ab+2ac+2bc+c^2+b^2$$

$$\frac{a+b+c}{3} < 0$$

$$a+b+c < 0$$

$$\sqrt{\frac{a^2+b^2+c^2}{3}} \geq 0$$

$$\Downarrow$$

$$\sqrt{\frac{a^2+b^2+c^2}{3}} \geq \frac{a+b+c}{3}$$

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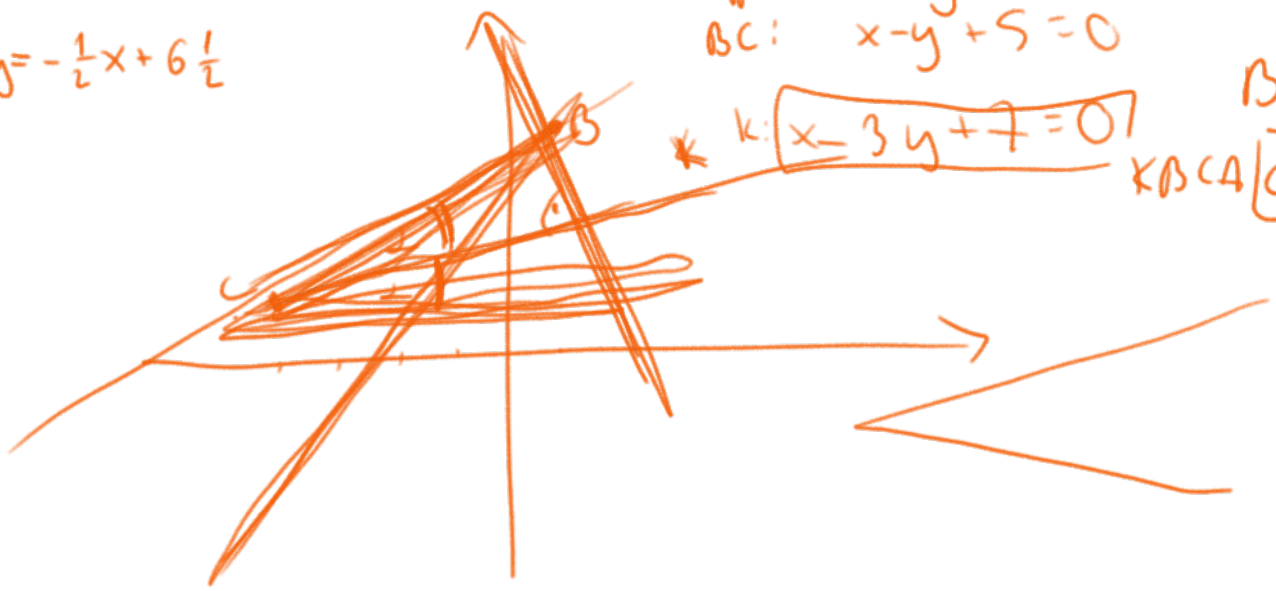
$y = -\frac{1}{2}x + 6\frac{1}{2}$



AB: $x + 2y - 13 = 0$

BC: $x - y + 5 = 0$

$B(1, 6)$

k: $x - 3y + 7 = 0$

$k \perp BC \quad A$

$C(-4, 1)$

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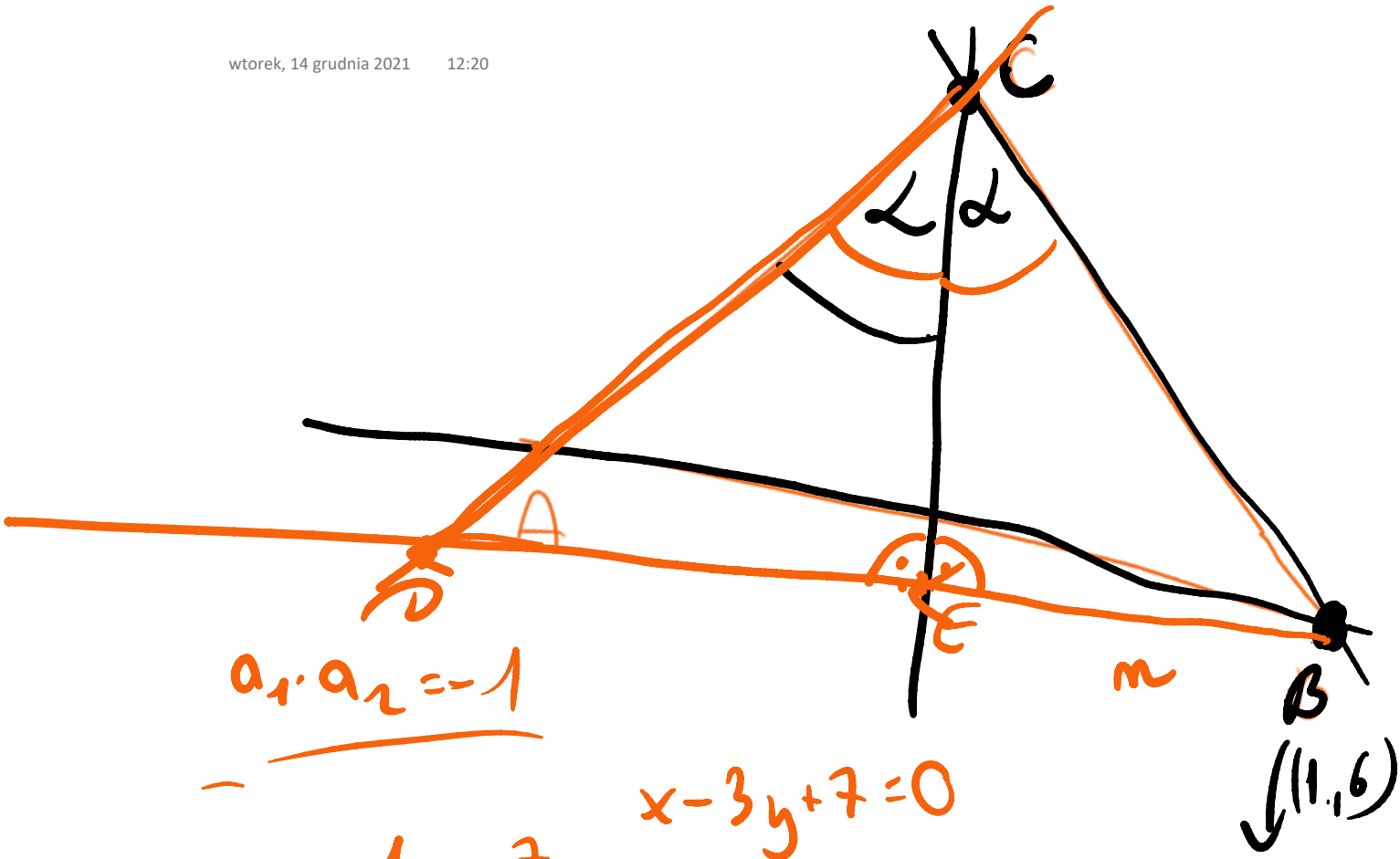

$a_1 \cdot a_2 = -1$

$x - 3y + 7 = 0$

$y = \frac{1}{3}x + \frac{7}{3}$

$y = a(x-p)+q$

$m: y = -3(x-1)+6$

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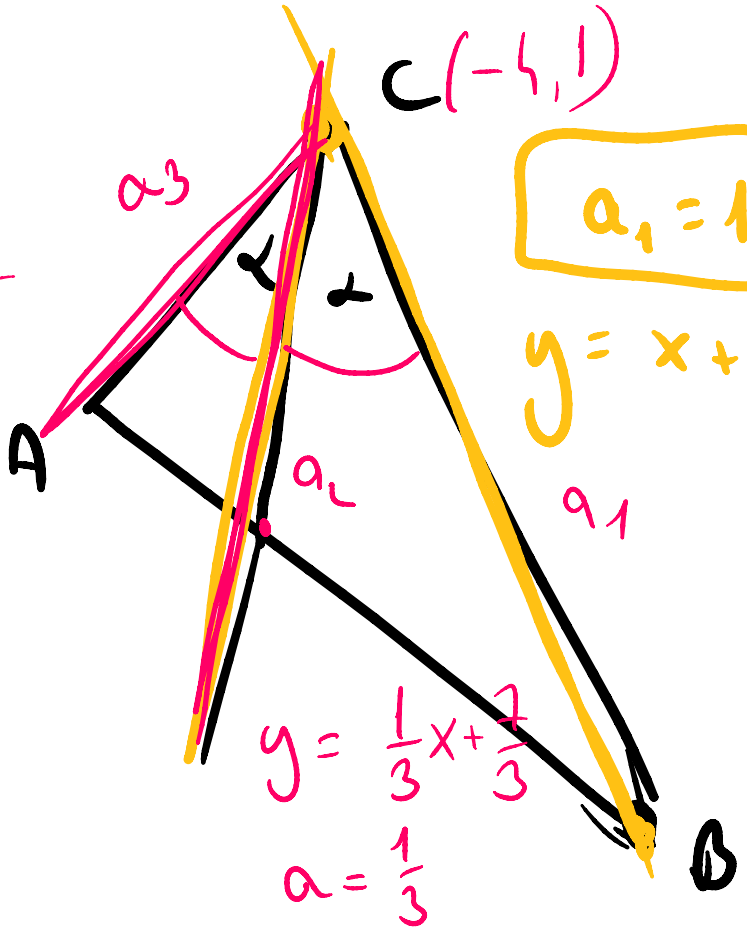
$y = a_3(x+4)+1$

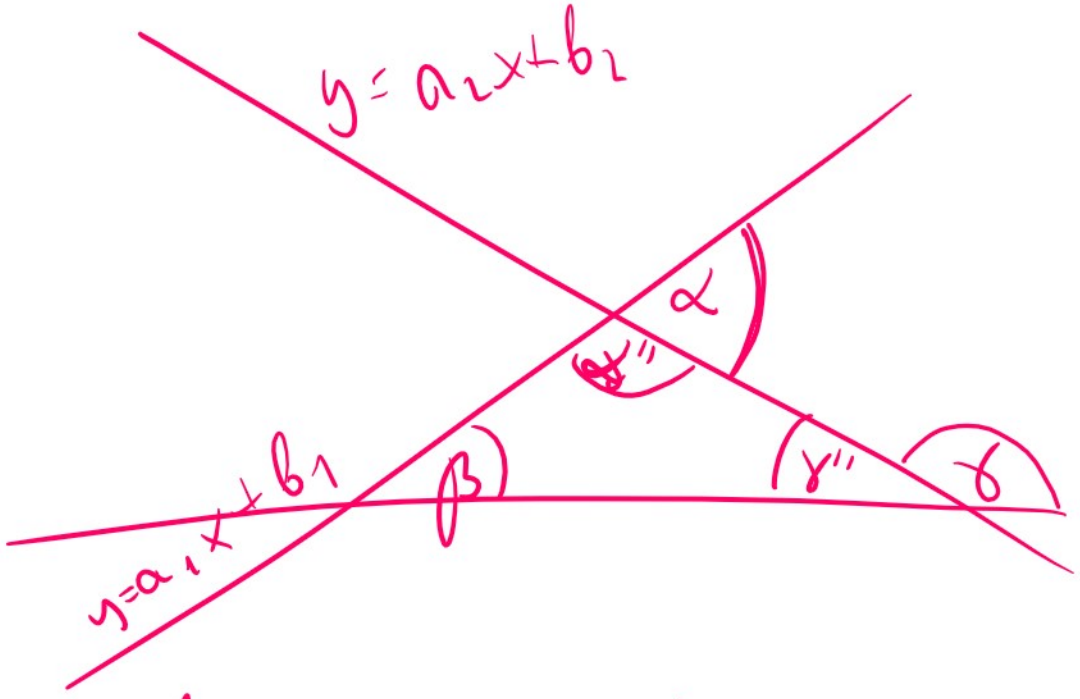


$$\operatorname{tg}\alpha = \left| \frac{a_1 - a_2}{1 + a_1 a_2} \right| = \left| \frac{a_3 - a_2}{1 + a_3 a_2} \right|$$

$$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha \operatorname{tg}\beta}$$

$a = \operatorname{tg}\alpha$

$y = ax + b$

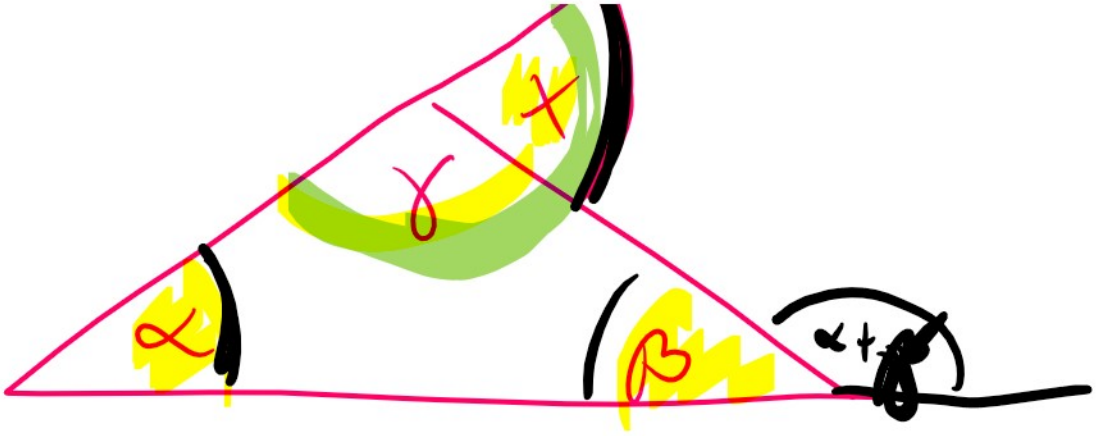


$\operatorname{tg}\beta = a_1 \qquad \operatorname{tg}\gamma = a_2$

$\alpha = 180 - \gamma + \beta$

$180 - \gamma + \beta + 180 - \alpha = 180$

$\boxed{\alpha = 180 - \gamma + \beta}$



$$\alpha + \beta = x$$

$$\alpha = 180 - \gamma + \beta$$

$$tg\alpha = tg(180 - \gamma + \beta) =$$

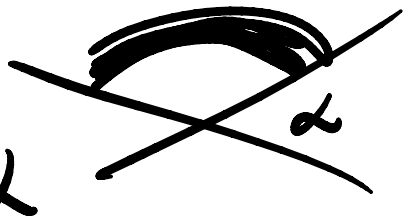
$$tg(180 + \beta - \gamma) = tg(\beta - \gamma)$$

$$tg(180 + \alpha)$$

$$\frac{tg\beta - tg\gamma}{1 + tg\beta \cdot tg\gamma} = \frac{a_1 - a_2}{1 + a_1 a_2}$$

$$\frac{\pi}{}$$

$\operatorname{tg}\alpha'' = \operatorname{tg}(180^\circ - \alpha) = -\operatorname{tg}\alpha$



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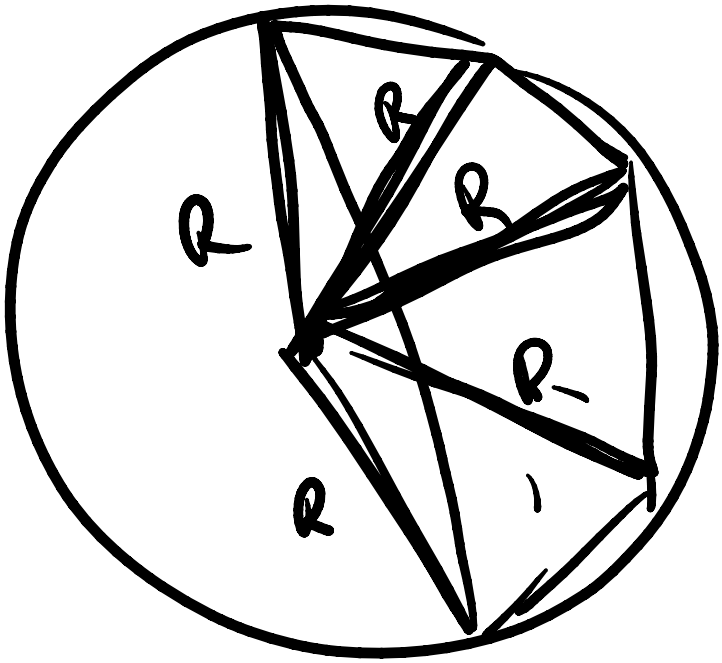
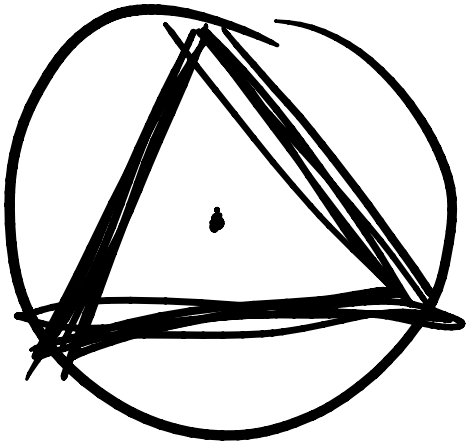
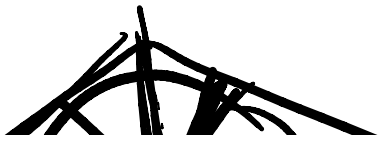




K ⇒ L ⇒ 3

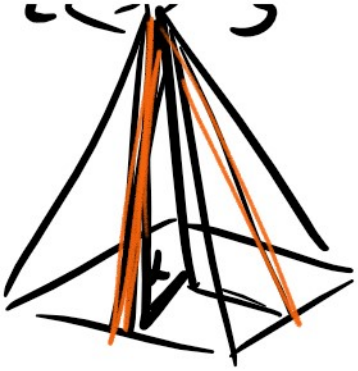
1) ⇔ 2 ⇒ 3

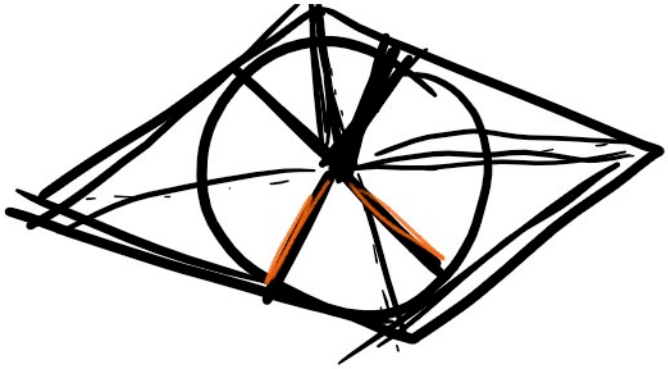


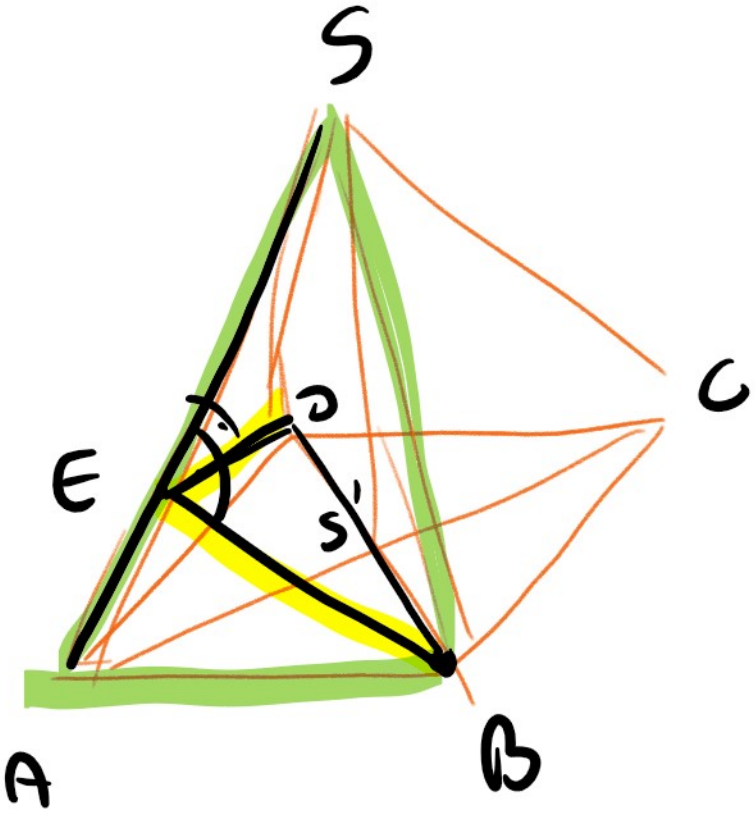
1)

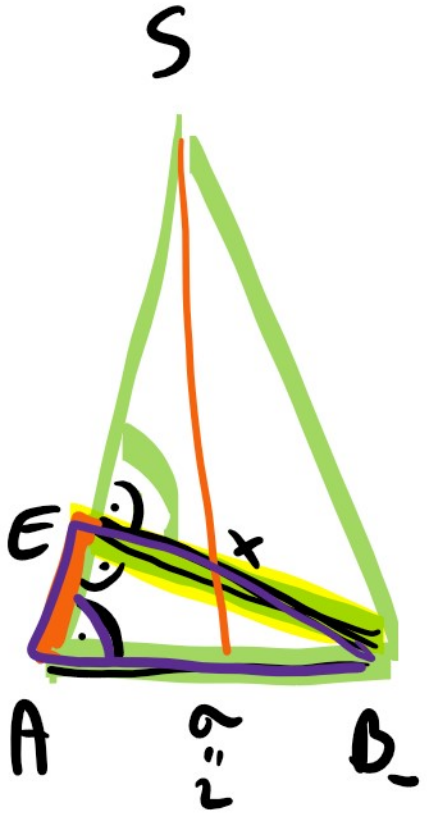
2)

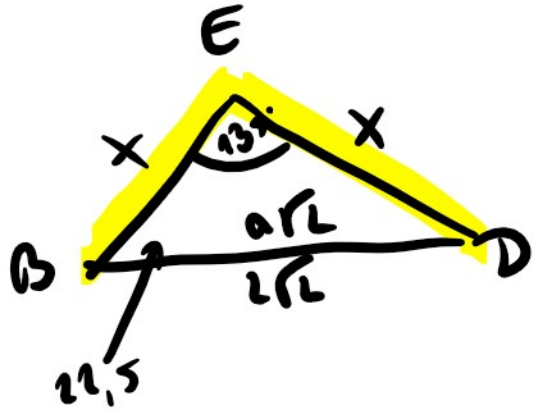
3)

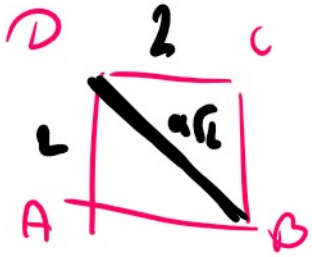












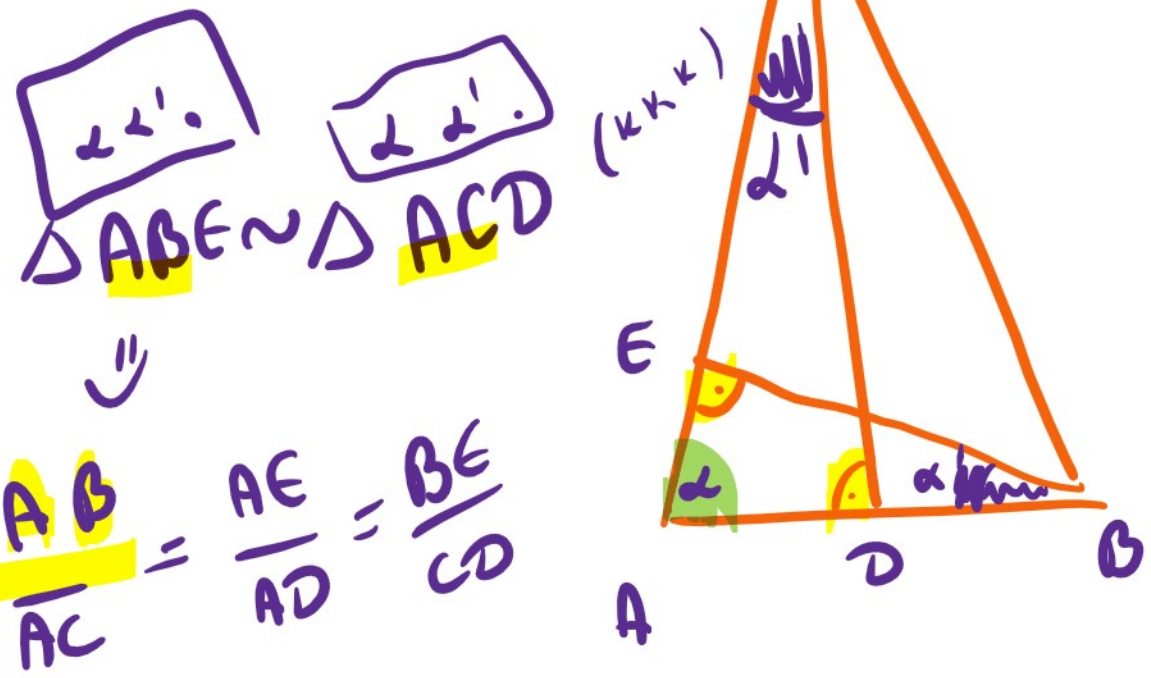
$y = \underbrace{2x^2 - 2x^2\cos 135^\circ}_{} = 2x^2 + \sqrt{2}\,x^2$

$x^2(2+\sqrt{2})$

$x > 0$

$x = \sqrt{8}$

$$x = \sqrt{\frac{8}{2+\sqrt{2}}}$$



$\triangle ABE \sim \triangle ACD$

$$\frac{AB}{AC} = \frac{AE}{AD} = \frac{BE}{CD}$$

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$|q|<1 \quad (x \neq 0)$

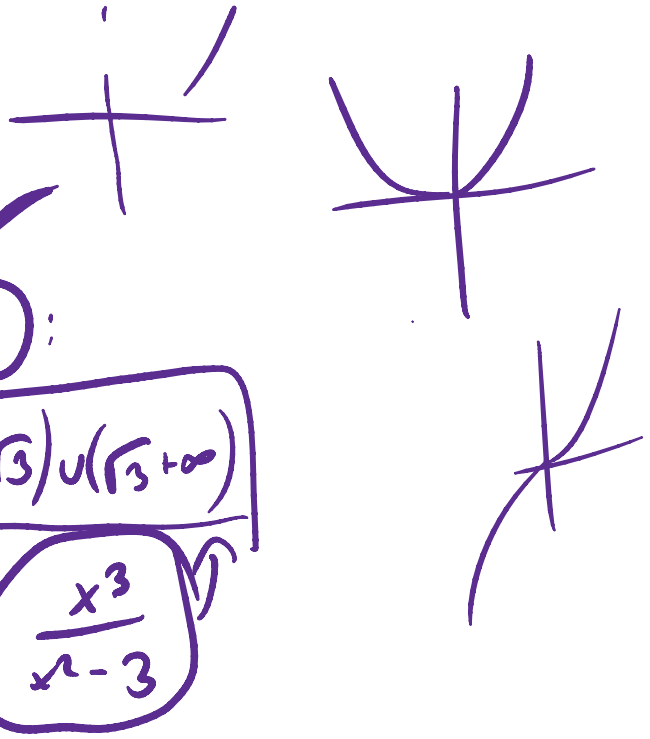
$|q| = \left|\frac{3}{x^2}\right| < 1 \quad / \cdot |x^2| > 0$

II:

$3 < x^2 \iff |x| > \sqrt{3}$

$x \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, +\infty)$

$f(x) = S = \frac{a_1}{1-q} = \frac{x}{1-\frac{3}{x^2}} = \frac{x^3}{x^2-3}$



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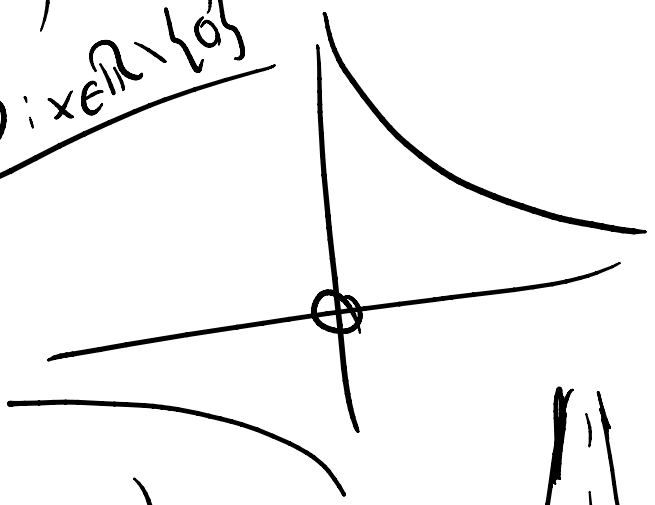
$D: x \in \mathbb{R} \setminus \{-1, -2\}$

$(-2, \dots)$

$\frac{(x-2)(x+2)}{(x+1)(x+2)(x^2-2x+4)}$

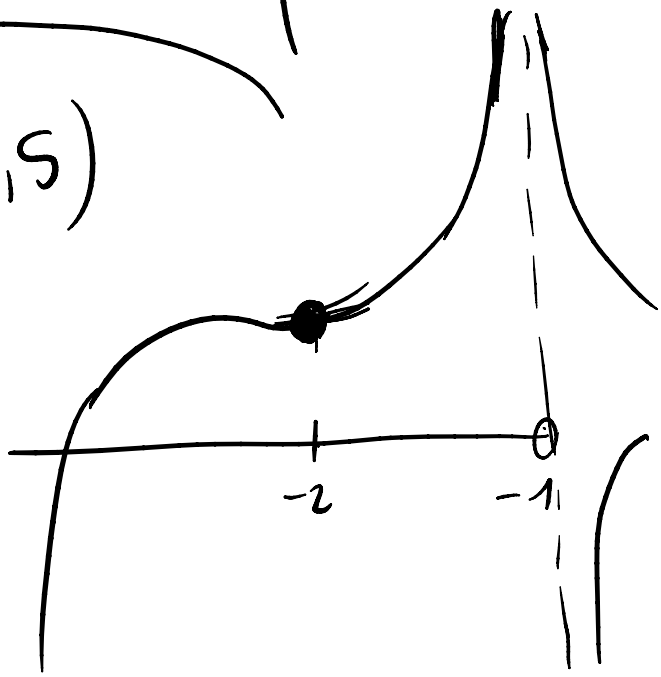
$y = \frac{1}{x}$

$D: x \in \mathbb{R} \setminus \{0\}$



$\lim_{x \to -2} f(x) = 5$ $(-2, 5)$

$\lim_{x \to -1} f(x) = \pm\infty$



$D: (-1, -2)$

$f(-2) = 5$

od -2

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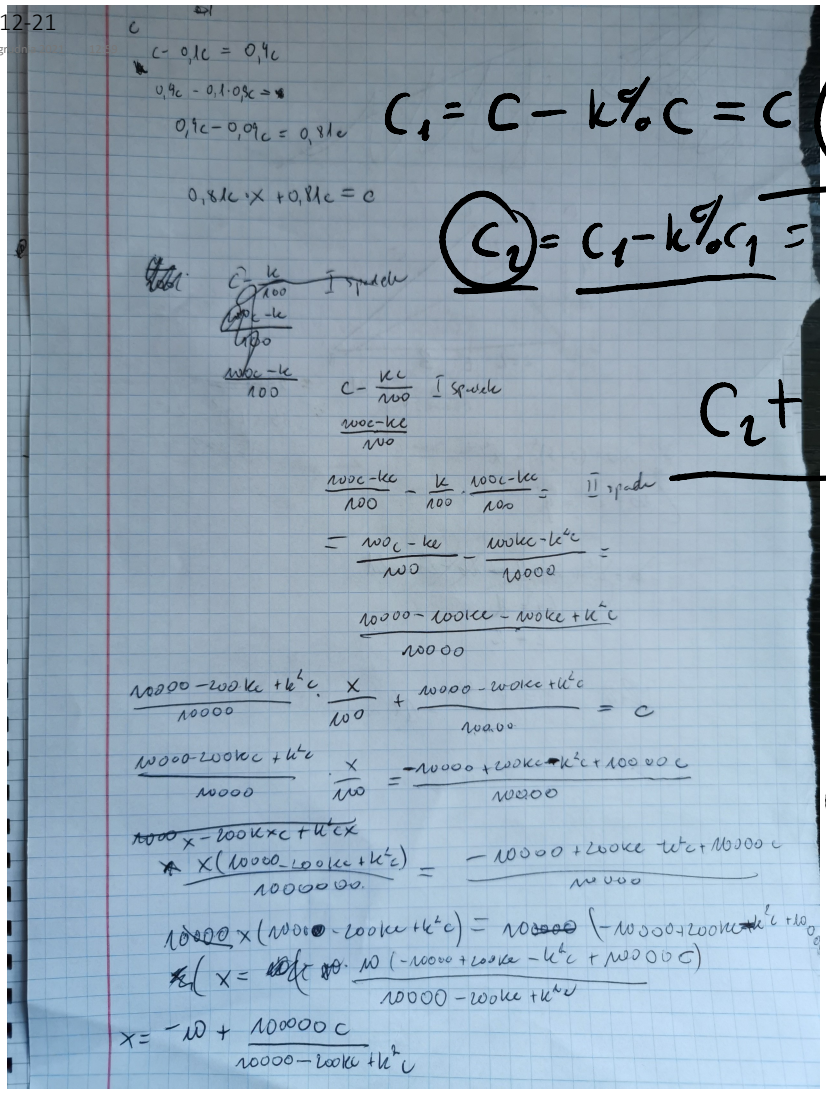

$C_1 = C - k\%\,C = C(1 - k\%)$

$C_2 = C_1 - k\%\,C_1 = C_1(1 - k\%) = C(1 - k\%)^2$

$C_2 + x\%\,C_2 = C$

$C_2(1 + x\%) = C$

$C(1 - k\%)^2 \cdot (1 + x\%) = C \mid :C \neq 0$

$(1 - k\%)^2 \cdot (1 + x\%) = 1$

$1 + \frac{x}{100} = \frac{1}{(1 - k\%)^2}$

$x = \frac{100}{(1 - k\%)^2}\% - 100$

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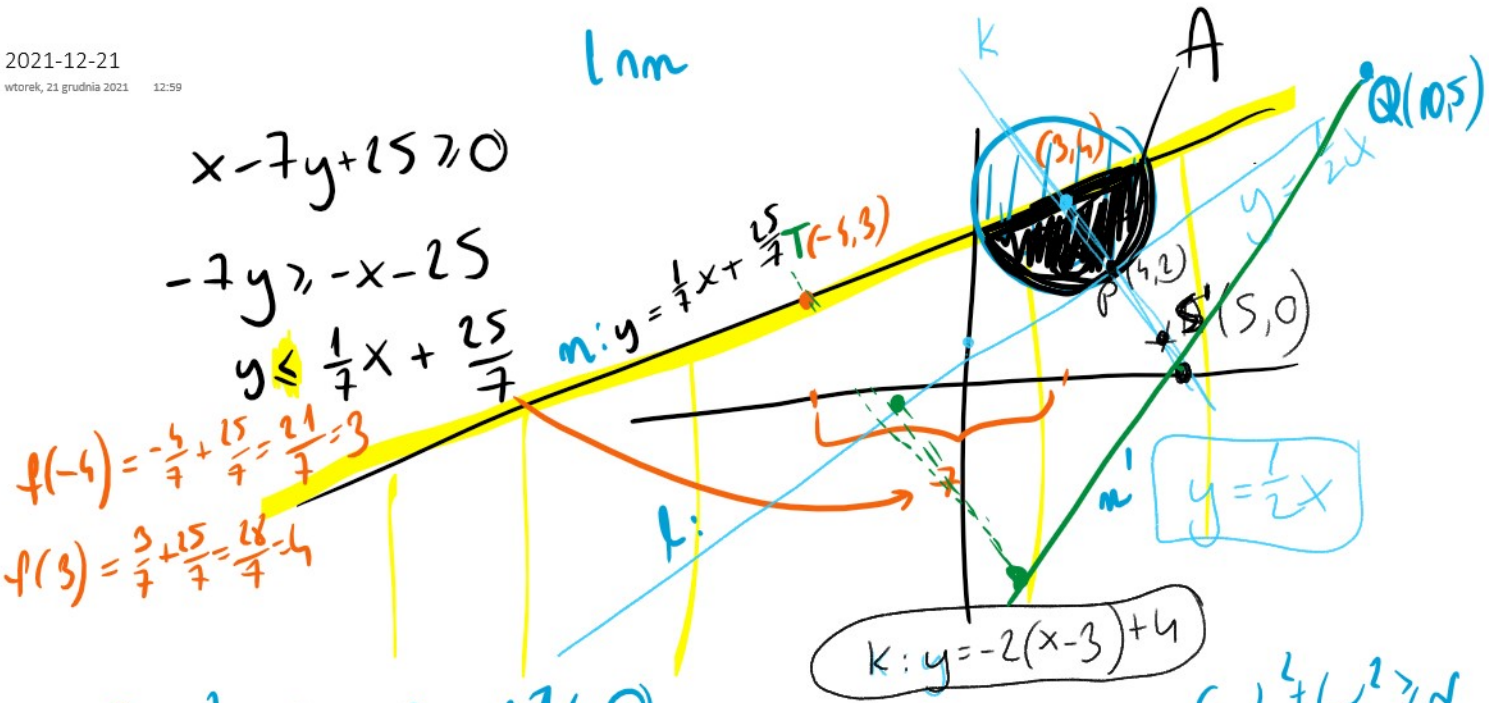

$x - 7y + 25 \geq 0$

$-7y \geq -x - 25$

$y \leq \frac{1}{7}x + \frac{25}{7}$

$x^2 + y^2 - 6x - 8y + 21 \leq 0$

$(x-3)^2 - 9 + (y-4)^2 - 16 + 21 \leq 0$

$(x-3)^2 + (y-4)^2 \leq 4$

$S(3,4)$ $r = 2$

$(3,4)$

$y = -2x + 10$

$y = \frac{1}{2}x$

$x = 4 \quad y = 2$

$-2x + 10 = \frac{1}{2}x$

$-2{,}5x = -10$

$x = 4 \quad y = 2$

$S'(5,0)$

$k': (x-5)^2 + y^2 \leq 4$

$\begin{cases} y = \frac{1}{7}x + \frac{25}{7} \\ \cdot \end{cases}$

$\begin{cases} x = 10 & Q(10,5) \\ y = 5 & S'(5,0) \end{cases}$

$y = x - 5$

$$\begin{cases} y = \frac{1}{2}x \end{cases}$$

$$\frac{1}{2}x - \frac{1}{7}x = \frac{25}{7}$$

$$\frac{5}{14}x = \frac{50}{14}$$

$$B := \{(x,y) : (x-5)^2 + y^2 \leq 4 \wedge y \geq x - 5\}$$

$$Obv = \frac{2\pi r}{2} + 2r$$

$$Obv = 2\pi + 4$$

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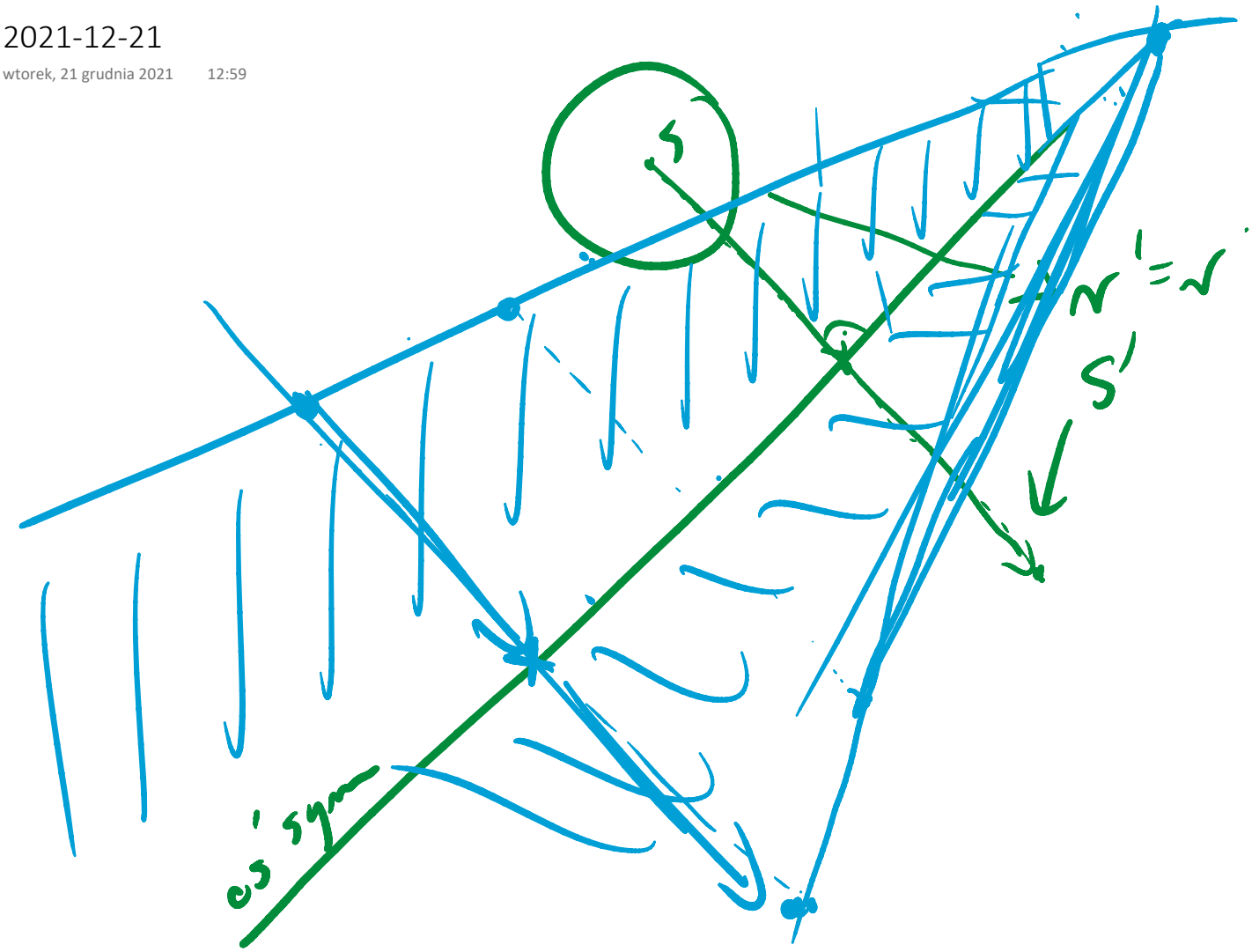

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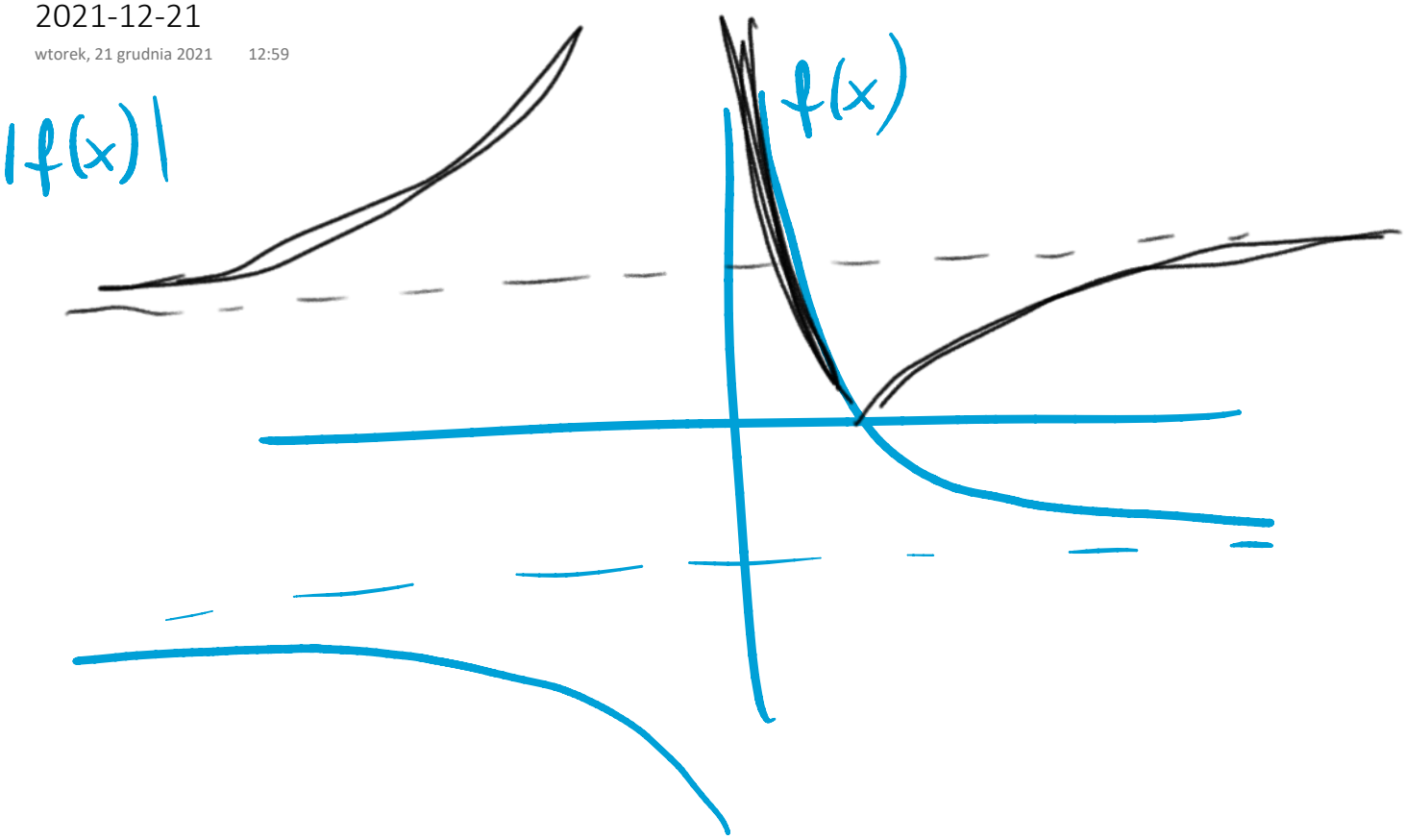

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$$\log_{2x}(x^4+3) \geq 2$$

$$D: \begin{cases} 2x>0 \Rightarrow x>0 \\ 2x \neq 1 \Rightarrow x \neq \frac{1}{2} \\ x^4+3>0 \Rightarrow x \in \mathbb{R} \end{cases} \Rightarrow x \in \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, +\infty\right)$$

$$\log_{2x}(x^4+3) \geq \log_{2x} 4x^2$$

$x \in \left(0, \frac{1}{2}\right)$ v $x \in \left(\frac{1}{2}, +\infty\right)$

$x^4+3 \leq 4x^2$

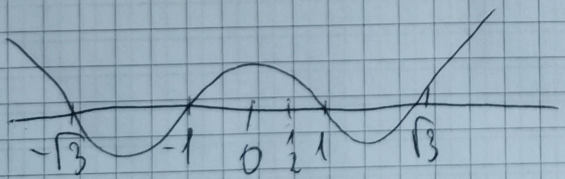
$x^4-4x^2+3 \leq 0$

$x^4-x^2-3x^2+3 \leq 0$

$x^2(x^2-1)-3(x^2-1) \leq 0$

$(x^2-1)(x^2-3) \leq 0$

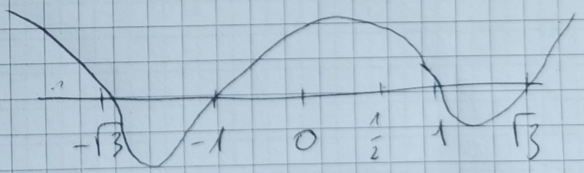
$(x-1)(x+1)(x-\sqrt{3})(x+\sqrt{3}) \leq 0$



$x \in \left(0, \frac{1}{2}\right) \cup (1, \sqrt{3}\rangle$

$x^4+3 \geq 4x^2$

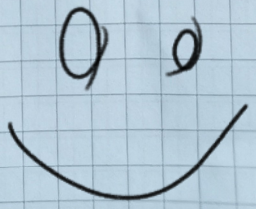
$(x-1)(x+1)(x-\sqrt{3})(x+\sqrt{3}) \geq 0$



$x \in \left(\frac{1}{2}, 1\right) \cup \left(\sqrt{3}, +\infty\right)$

$x \in \emptyset$

$x \in \left(\frac{1}{2}, 1\right) \cup \left(\sqrt{3}, +\infty\right)$



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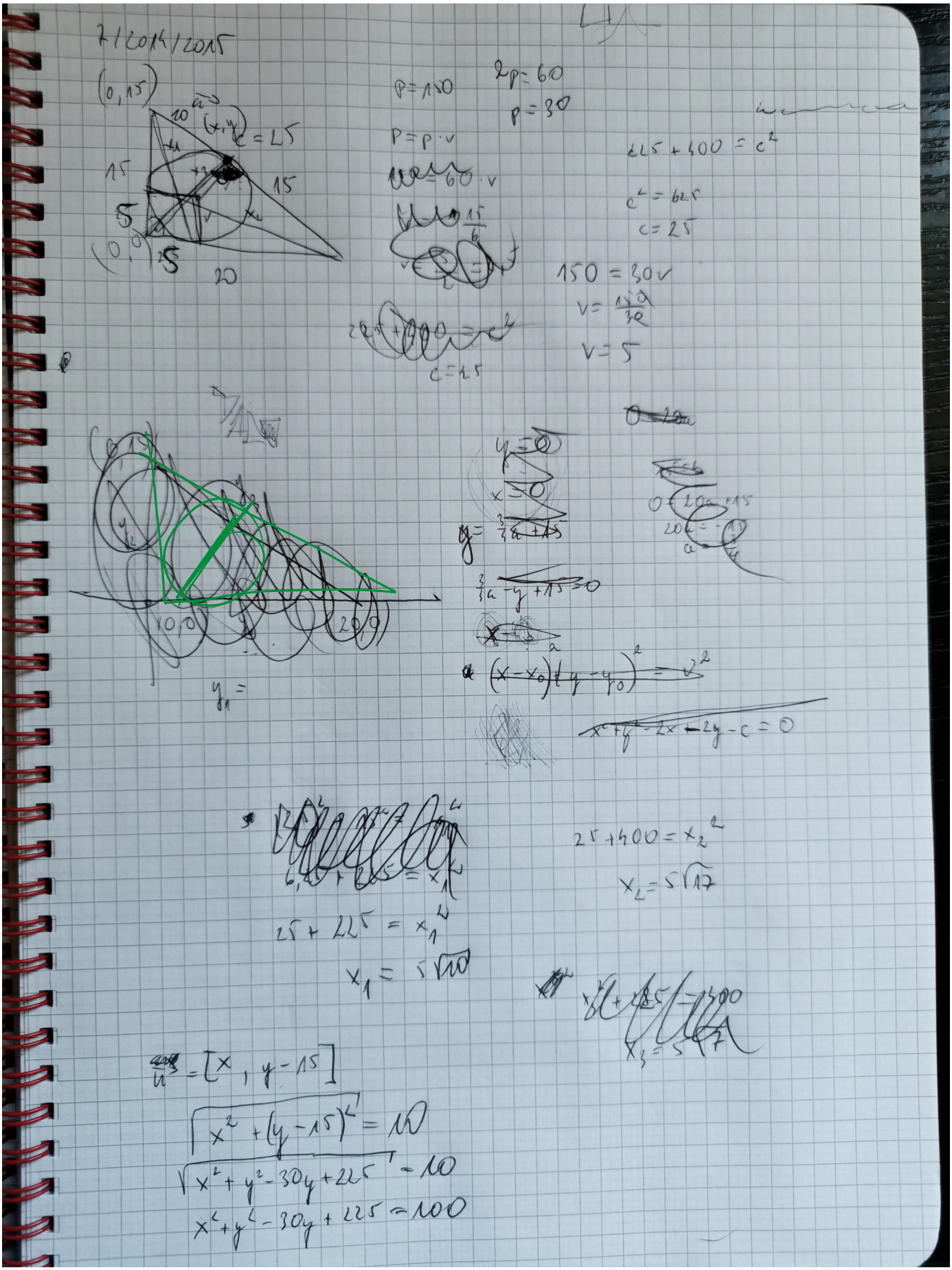
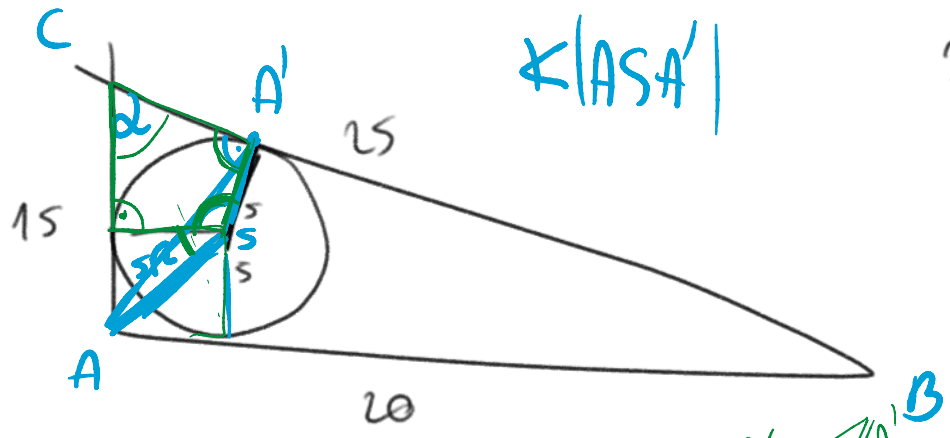
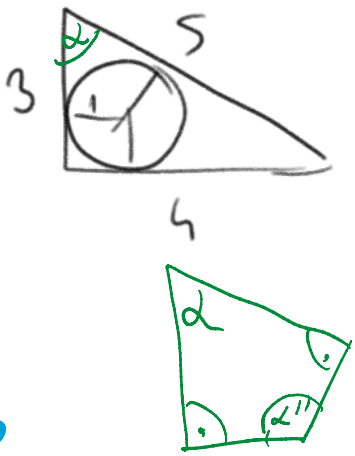
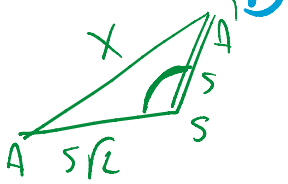




$\measuredangle(ASA')$





$|\measuredangle ASA'| = \alpha'' + 45^\circ$

$\cos(\alpha'' + 45^\circ) = \cos\alpha''\cos 45^\circ - \sin\alpha''\sin 45^\circ = (-\cos\alpha - \sin\alpha)\frac{\sqrt{2}}{2}$

$\cos(\alpha'' + 45) = \left(-\frac{3}{5} - \frac{4}{5}\right)\frac{\sqrt{2}}{2}$

$\frac{-7\sqrt{2}}{10}$

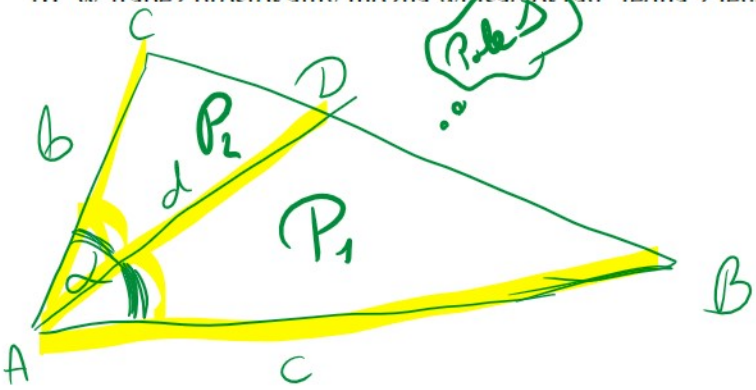
$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos(\alpha+\alpha) = \cos\alpha\cdot\cos\alpha - \sin\alpha\cdot\sin\alpha$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

9. W trójkącie ostrokątnym ABC bok AB ma długość c, długość boku AC jest równa b oraz $|\angle BAC| = \alpha$. Dwusieczna kąta BAC przecina bok BC trójkąta w punkcie D i odcinek AD ma długość d. Wykaż, że $\cos\frac{\alpha}{2} = \frac{d}{2b} + \frac{d}{2c}$.

10. W trapez prostokątny można wpisać okrąg. Jedna z jego podstaw ma długość a, druga jest trzy razy



Rok D

$P_1 + P_2 = \frac{1}{2}cb \cdot \sin\alpha =$

$\frac{1}{2}bd\sin\frac{\alpha}{2} + \frac{1}{2}cd\sin\frac{\alpha}{2} \quad /\cdot 2$

$/\cdot \sin\frac{\alpha}{2}$

$cb\frac{\sin\alpha}{\sin\frac{\alpha}{2}} = bd + cd \quad /:cb$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\sin\alpha = 2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}$

$\frac{2\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} = \frac{d}{c} + \frac{d}{b} \quad /\cdot\frac{1}{2}$

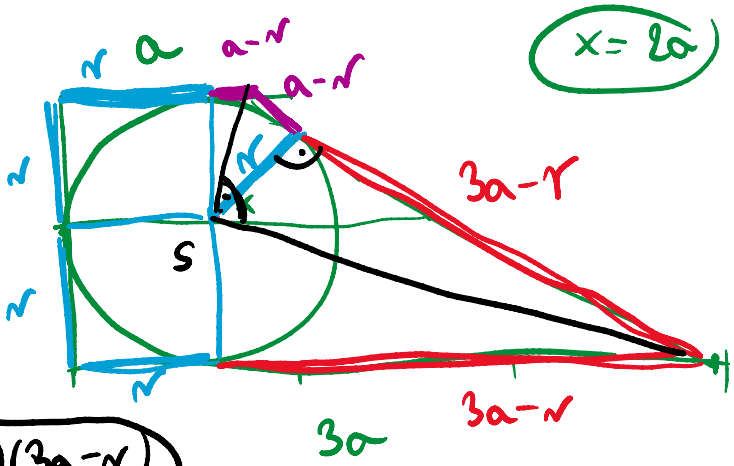
$\cos\frac{\alpha}{2} = \frac{d}{2c} + \frac{d}{2b}$

$h = 2r$ $\quad P = \frac{a+b}{2} \cdot h = 2a \cdot h$ $\quad h = ?$

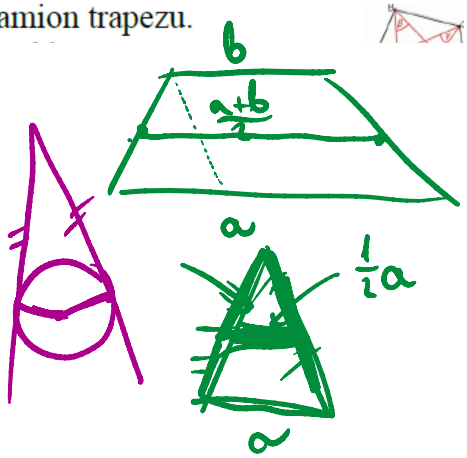
$2 \quad 2b \quad 2c$

10. W trapez prostokątny można wpisać okrąg. Jedna z jego podstaw ma długość a, druga jest trzy razy dłuższa. Oblicz pole trapezu oraz długość odcinka łączącego środki ramion trapezu.

Dane

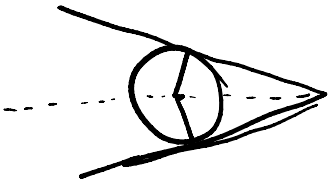


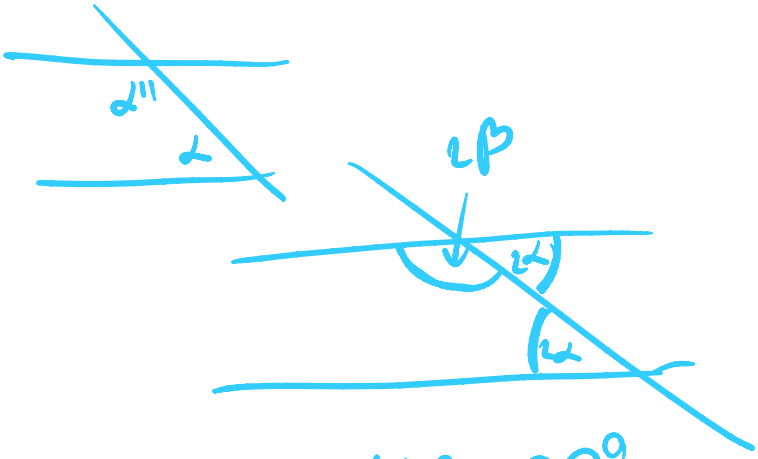
$x = 2a$



z TU

$r^2 = (a-r)(3a-r)$



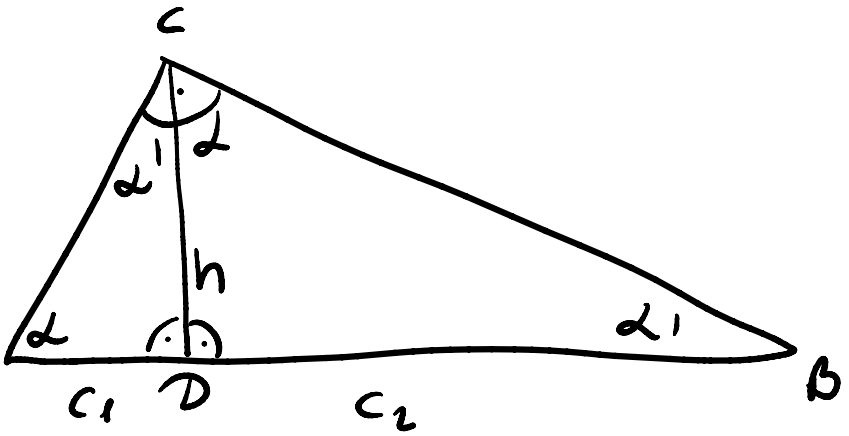


$\alpha + \beta = 90^\circ$

ΔBCS

$2\alpha + 2\beta = 180$

$h^2 = c_1 \cdot c_2$



$\alpha \cdot \alpha'$

$\Delta ADC \sim \Delta CDB \Rightarrow \frac{AD}{CD} = \frac{CD}{DB} \Leftrightarrow CD^2 = AD \cdot BD$