Zadanie 14.16. [matura, maj 2018, zad. 15. (7 pkt)]

Rozpatrujemy wszystkie trapezy równoramienne, w które można wpisać okrąg, spełniające warunek: suma długości dłuższej podstawy a i wysokości trapezu jest równa 2.

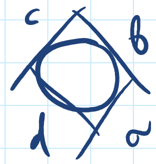
a) Wyznacz wszystkie wartości a, dla których istnieje trapez o podanych własnościach.

b) Wykaż, że obwód L takiego trapezu, jako funkcja długości a dłuższej podstawy trapezu, wyraża się wzorem $L(a) = \frac{4a^2 - 8a + 8}{a}$.

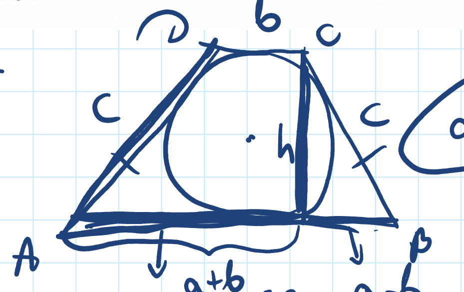
c) Oblicz tangens kąta ostrego tego spośród rozpatrywanych trapezów, którego obwód jest najmniejszy.

$a > h \mid L = a + h > 2h$

$h < 1$ $\quad a > 1$



$a + c = b + d$



$a > 0$

$h > 0$

I) $a > 1$

$a + h = 2$

$r_0 = \frac{1}{2} h$

$a \in (1, 2)$

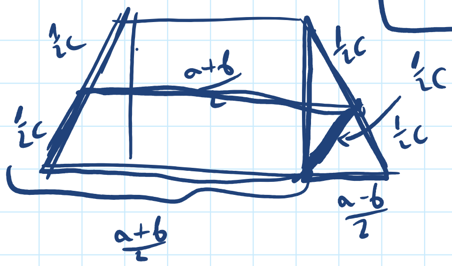
$\frac{a+b}{2} = c$ $\quad \frac{a-b}{2}$

$a + b = 2c$

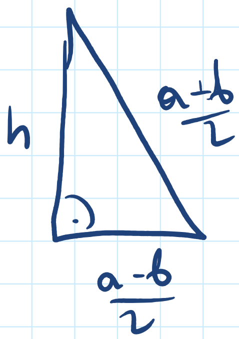
$\alpha + \gamma = \beta + \delta$

PTOLEMEUSZA

$a \cdot c + b \cdot d = d_1 d_2$



$L = a + b + 2c = 2(a + b)$



$h^2 = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 =$

$\boxed{h^2 = ab}$

$h^2 = (2-h)b$

$b = \frac{h^2}{2-h} = \frac{(2-a)^2}{2-2+a} =$

$\boxed{h = 2 - a}$

$2 - h$

$\downarrow$

$L = 2(a+b) = 2\left(\underline{2-h} + \frac{h^2}{2-h}\right) =$

$L(a) =$

$$\boxed{b = \frac{h^2}{2-h} \longleftarrow h = 2 - a}$$

$L = \frac{(2-a)^2}{}$

$$b = \frac{(1-a)^2}{}$$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - g' f}{g^2}$$

1.15. Oblicz granice:

a) $\lim_{x\to 7}\frac{\sin\sqrt{x}-\sin\sqrt{7}}{2x-14}$; b) $\lim_{x\to 0}\frac{x-\sin 2x}{x-\sin 5x}$;

*c) $\lim_{x\to 0}\frac{\cos(x+x^2)-1}{\sin x^2}$; d) $\lim_{x\to\frac{\pi}{3}}\frac{\operatorname{tg}^3 x-3\operatorname{tg} x}{\cos\left(x+\frac{\pi}{6}\right)}$;

e) $\lim_{x\to\frac{\pi}{4}}\frac{1-\operatorname{ctg}^3 x}{2-\operatorname{ctg} x-\operatorname{ctg}^3 x}$; f) $\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$;

*g) $\lim_{x\to 0}\frac{x^2}{\sqrt{1+x\sin x}-\sqrt{\cos x}}$; *h) $\lim_{x\to+\infty}\left(\sin\sqrt{x+1}-\sin\sqrt{x}\right)$.

a) $\lim_{x\to 7}\frac{\sin\sqrt{x}-\sin\sqrt{7}}{2x-14}$;

$\sqrt{x}$ $\quad x \geq 0$

$x > 0$

$x \to 7 \quad / -7$

$x-7 \to 0$

$\sqrt{x} \to \sqrt{7}$

$\sqrt{x}-\sqrt{7} \to 0$

$\frac{\sqrt{x}-\sqrt{7}}{2} \to 0$

$\lim_{\alpha\to 0}\frac{\sin a\alpha}{b\alpha} = \frac{a}{b}$

$\frac{\cos\sqrt{7}}{2}.$

b) $\lim_{x\to 0}\frac{x-\sin 2x}{x-\sin 5x}$; $= \lim_{x\to 0}\frac{1-\frac{\sin 2x}{x}}{1-\frac{\sin 5x}{x}}$

$\frac{\sin 2x}{x} \to 2$

$\frac{\sin 5x}{x} \to 5$

*c) $\lim\limits_{x\to 0}\dfrac{\cos\left(x+x^2\right)-1}{\sin x^2}$;   d) $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\operatorname{tg}^3 x-3\operatorname{tg}x}{\cos\left(x+\frac{\pi}{6}\right)}$;

e) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\operatorname{ctg}^3 x}{2-\operatorname{ctg}x-\operatorname{ctg}^3 x}$;   f) $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$;

*g) $\lim\limits_{x\to 0}\dfrac{x^2}{\sqrt{1+x\sin x}-\sqrt{\cos x}}$; *h) $\lim\limits_{x\to+\infty}\left(\sin\sqrt{x+1}-\sin\sqrt{x}\right)$.

$1 = \cos 0^\circ$

$$\lim_{x\to 0}\frac{\cos(x+x^2)-1}{\sin x^2}$$

$$\lim_{x\to 0}\frac{x^2}{\sin x^2}\to\frac{1}{1}=1$$

$\to 1$

$$\frac{\sin ax}{\sin bx}\cdot\frac{bx}{bx}\cdot\frac{ax}{ax}$$

f) $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$;

$$\frac{(\sin x+1)\cancel{(2\sin x-1)}}{(\sin x-1)\cancel{(2\sin x-1)}}$$

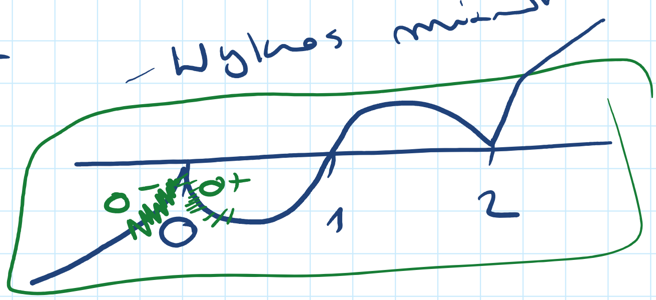
1.18. Znajdź granice jednostronne funkcji $y=f(x)$ w punktach $x_0=0$, $x_0=1$, $x_0=2$, jeśli:

a) $f(x)=\dfrac{|x|(x+2)}{x(x-1)^3(x-2)^2}$; b) $f(x)=\dfrac{x(x+1)}{(x-1)^2(x+2)}$; c) $f(x)=\dfrac{x^2-4}{|x-2|x^3(x-1)^2}$.

$$\lim_{x\to 0^-}\frac{(x-2)(x+2)}{|x-2|\,x^3(x-1)^2}$$

$$L(0)=\left[\frac{-4}{0^-}\right]=+\infty$$

- Wykres mi[illegible]



$L(0) = \left[\frac{}{0^-}\right]$

$P(0) = \left[\frac{-5}{0^-}\right] = +\infty \qquad L(0) = P(0) = \lim_{x \to 0} f(x) = +\infty$

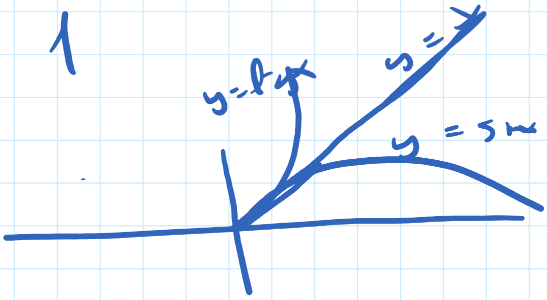
wartości małe

$L(1) = \lim_{x \to 1^-} f(x) = \left[\frac{-3}{0^-}\right] =$

$x \to 2^- \qquad x < 2 \qquad |\overset{-}{x-2}| = -(x-2)$

$x \to 2^+ \qquad x > 2 \qquad |x-2| = (x-2)$

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$



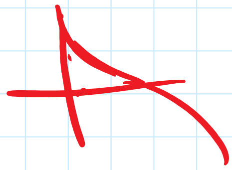
$$\lim_{x\to 0}\frac{tg\,x}{x}=1$$

$$\boxed{\sin x \le x \le tg\,x}$$
$x\to 0$

$$\lim_{x\to 0}\frac{\cos x}{x}=\left[\frac{\cos 0}{0}=\frac{1}{0}=\pm\infty\right]$$

NIE ISTNIEJE

$$\lim_{x\to 0}\frac{ctg\,x}{x}=+\infty \qquad \frac{+\infty}{0^+} \qquad \frac{-\infty}{0^-}$$



$$\lim_{x\to 0}\frac{\sin ax}{bx}=\frac{a}{b}$$

$$\lim_{x\to 0}\frac{\sin ax}{\sin bx}=\frac{a}{b}=$$

$$\lim_{x\to 0}\frac{\sin ax}{tg\,bx}=\frac{a}{b}$$

$$\underbrace{\lim\frac{f(x)}{g(x)}}_{}\ \left(\frac{\infty}{\infty}\quad\frac{0}{0}\right) \overset{H}{=} \lim\frac{f'(x)}{g'(x)}$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\boxed{x \approx 0 \Rightarrow \sin x \approxeq x \approxeq \operatorname{tg} x}$$

$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + \dots + b^{n-1})$

c) $\lim\limits_{x\to 0} \dfrac{x^2}{\sqrt[5]{1+5x}-(1+x)}$;

d) $\lim\limits_{x\to 0} \dfrac{\sqrt[n]{1+x}-1}{x}$ $(n \in N_+)$;

e) $\lim\limits_{x\to 0} \dfrac{\sqrt[m]{1+\alpha x}\cdot\sqrt[n]{1+\beta x}-1}{x}$ $(m,n \in N_+, \alpha,\beta \in R)$;

$\sqrt[5]{a^4} = a^{\frac{4}{5}}$

$(1+5x)^{\frac{4}{5}} + (\quad)^{\frac{3}{5}}$

$\dfrac{1}{a-b} = \dfrac{a^4 + a^3b + a^2b^2 +}{(a^5 - b^5)}$

$1 + 5x - 1 - 5x$

$(a+b)^5$ $\Rightarrow 1\ 2\ 1$ $(a+b)^2$

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1