Z: $f(x)=\dfrac{2x-1}{x-3}$

D: $x\in\mathbb{R}\setminus\{3\}$

$x\in(-\infty,3)\cup(3,+\infty)$

$x_1-x_2<0$

T: $f(x_1)-f(x_2)$?

D: $f(x_1)-f(x_2)=\dfrac{2x_1-1}{x_1-3}-\dfrac{2x_2-1}{x_2-3}=\dfrac{(2x_1-1)(x_2-3)}{(x_1-3)(x_2-3)}-\dfrac{(2x_2-1)(x_1-3)}{(x_1-3)(x_2-3)}$

$$=\frac{2x_1x_2-6x_1-x_2+\cancel{3}-(2x_2x_1-6x_2-x_1+\cancel{3})}{(x_1-3)(x_2-3)}=\frac{-5x_1+5x_2}{\underbrace{(x_1-3)(x_2-3)}_{\text{PIĘKNE}}}=$$

$$=\frac{-5(x_1-x_2)}{(x_1-3)(x_2-3)}=\frac{\pm}{+}>0\Rightarrow$$

$x\in(-\infty,3)$ $-$ $-$

$(3,+\infty)$ $+$ $+$

$M>0$ dla $x\in(-\infty,3)$ $f\downarrow$

$M>0$ dla $x\in(3,+\infty)$ $f\downarrow$

$x\in(-\infty,3)\cup(3,+\infty)\Rightarrow$ M nie ma określonego znaku. musi być dodatni w ujęciu.

$\boxed{x_1-x_2<0}\Rightarrow\boxed{f(x_1)-f(x_2)<0}$ $f\nearrow$

3 - 4   4   4,5

$f(x_1)-f(x_2)>0$ $f\downarrow$

$x\in(-\infty,3)\Rightarrow f\downarrow$

$x \in (3, +\infty) \Rightarrow f \searrow$

$$\boxed{f(x) = -\sqrt{4-3x} - 2}$$

$4 - 3x \geq 0$

$-3x \geq -4 \quad | : (-3)$

$x \leq \frac{4}{3}$

D:

Z: $\underbrace{f(x) = -\sqrt{4-3x} - 2}_{\text{ROSNĄ}}$

D: $\underbrace{x \in \left(-\infty, 1\tfrac{1}{3}\right\rangle}_{x_1 \quad x_2}$

ROSNĄ

$\underbrace{x_1 - x_2 < 0}_{\Downarrow}$

$x_1 \neq x_2$

T: $\underline{f(x_1) - f(x_2) < 0}$

D: $f(x_1) - f(x_2) = -\sqrt{4-3x_1} - 2 - \left(-\sqrt{4-3x_2} - 2\right) =$

$+2$

3.113
3.114 d
3.115 bd

$= \underbrace{-\sqrt{4-3x_1}}_{-} + \underbrace{\sqrt{4-3x_2}}_{+} = \underbrace{\sqrt{4-3x_2} - \sqrt{4-3x_1}}_{+ - a} =$

$$\sqrt{a} - \sqrt{b} = \frac{a-b}{\sqrt{a}+\sqrt{b}}$$

$$= \frac{(\cancel{4} - 3x_2) - (\cancel{4} - 3x_1)}{\sqrt{4-3x_2} + \sqrt{4-3x_1}} = \frac{3x_1 - 3x_2}{\sqrt{4-3x_2} + \sqrt{4-3x_1}} \; +$$

$\underbrace{\geq 0 \neq \cdot \geq 0}_{> 0}$

$\sqrt{4-3x_1} = 0$

$x_1 = \frac{4}{3}$

$x_2 = \frac{4}{3}$

$+ \cdot -$

$$\frac{3(x_1 - x_2)}{\sqrt{4-3x_2} + \sqrt{4-3x_1}} = \frac{-}{+} < 0 \Rightarrow$$

$+$

WZROST

WZROST

$x_1 - x_2 < 0 \Rightarrow f(x_1) - f(x_2) < 0 \Rightarrow$ <u>ROSNĄCA</u>

$$(a-b)(a+b)=a^2-b^2$$

$$\Updownarrow$$

$$(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})=a-b \quad \Big| :(\sqrt{a}+\sqrt{b})$$

$a,b \in \mathbb{R}_+$

$$\boxed{\sqrt{a}-\sqrt{b}}=\frac{a-b}{\sqrt{a}+\sqrt{b}}$$

$\sqrt{a}+\sqrt{b}>0$

$+ \; - \; 0$?

$+$

2022-01-31-funkcje

poniedziałek, 31 stycznia 2022 08:57

x	1	2	3	4	5	6
y	-1	-2	-3	-4	-5	-6

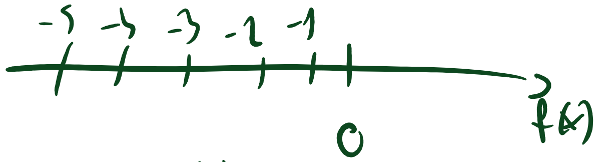
$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6$

$x_1 - x_2 < 0$ ciąg rosnący

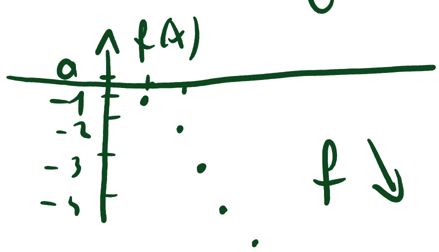
$\boxed{x_4 - x_6 < 0}$

ciąg malejących wartości

$\boxed{f(x_1) - f(x_2) > 0}$ $f \downarrow$

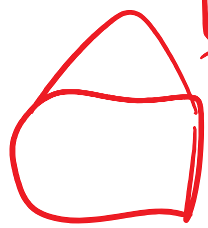


$f(x) = -x$ $f \downarrow$



$f \downarrow$

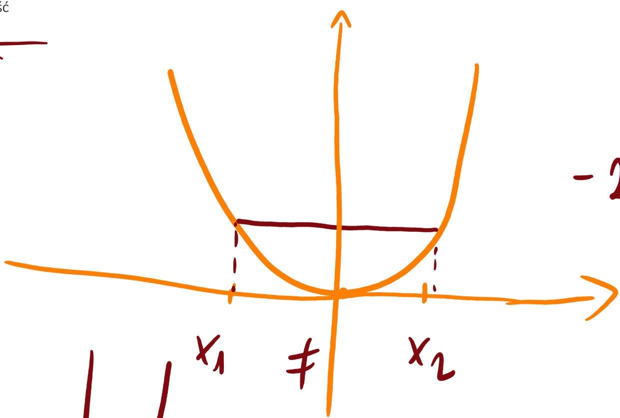
ZMIANA ZNAKÓW NA PRZECIWNE



D

zad 109 - 115

Ćwiczenia 105 - 108



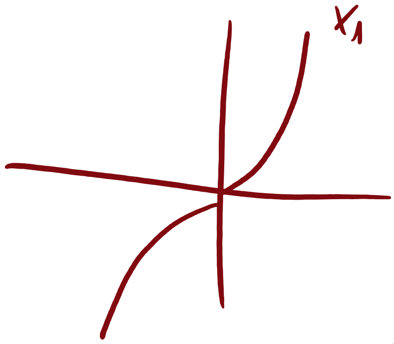
$f(x) = x^2$

$-2 = x_1 \neq x_2 = 2$

$f(-2) = 4$

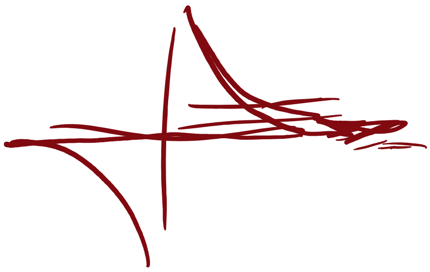
$f(2) = 4$

$f(-2) = f(2)$

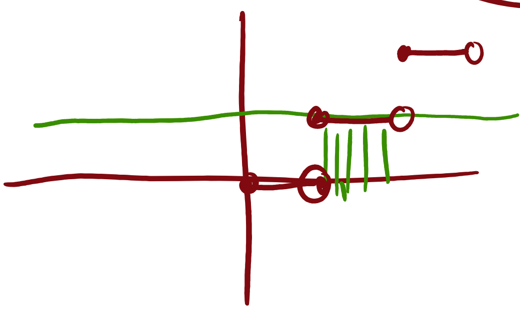


$y = \frac{1}{x}$

$y = \frac{1}{x}$



$y = [x]$



nie jest różn

$f(x) = [x]$

$f(1,1) = [1,1] = 1$

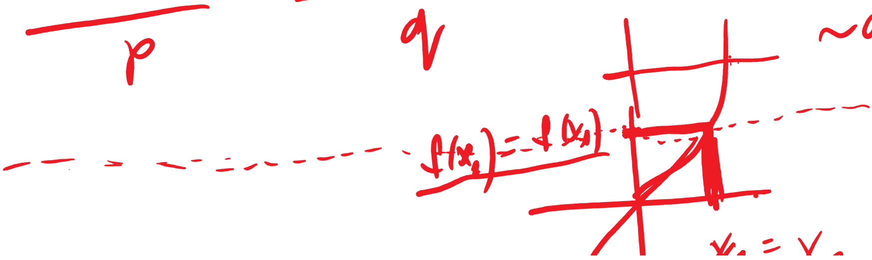
$f(1,2) = [1,2] = 1$

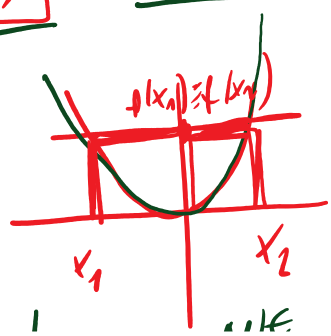
$\boxed{x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)} \Rightarrow f$ nie jest różnowartościowa

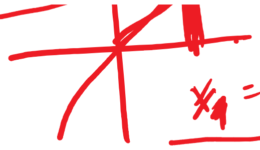
$f: X \to Y$. f różnowartościowa $\overset{def}{\Longleftrightarrow} \forall_{x_1, x_2 \in \mathbb{R}}\ x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

$p \Rightarrow q \Leftrightarrow \sim q \Rightarrow \sim p$

$\underbrace{(x_1 \neq x_2)}_{p} \Rightarrow \underbrace{f(x_1) \neq f(x_2)}_{q} \Leftrightarrow \underbrace{\boxed{f(x_1) = f(x_2)}}_{\sim q} \Rightarrow \underline{x_1 = x_2}$



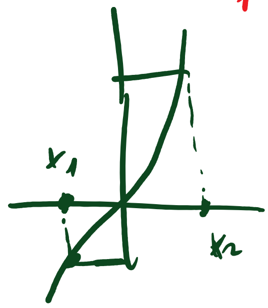




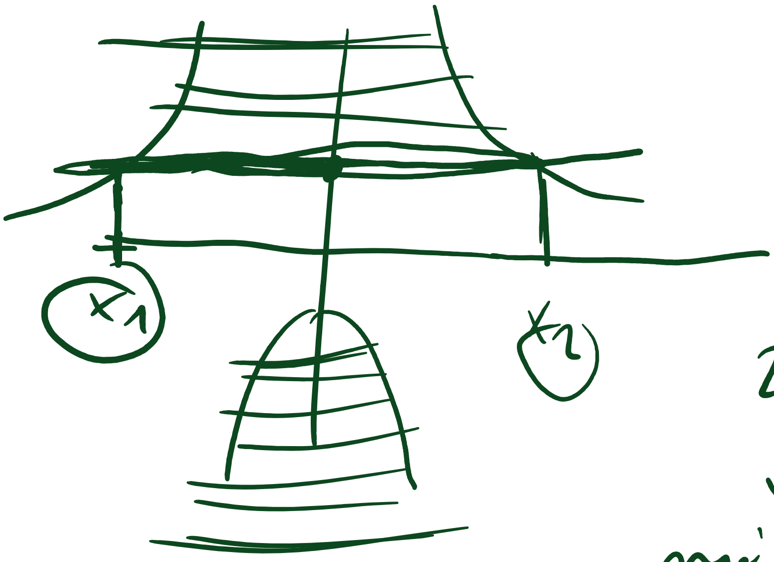
$x_1 = x_2$

x_1 | x_2

NIE



TAK

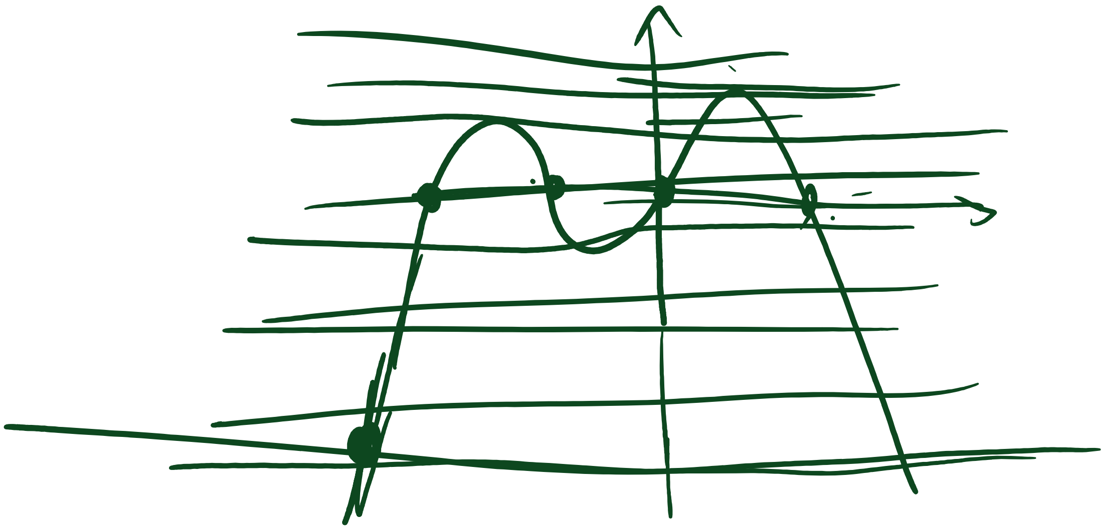


$x_1 \neq x_2$

$f(x_1) = f(x_2)$

2 razy p

nie jest r.



3.125f $f(x)=\frac{\sqrt{3}x-2}{4x-3}$ $D: x\in\mathbb{R}\setminus\{\frac{3}{4}\}$

DEF

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ } dobre do pokazania, że nie jest (R)

$\Updownarrow$

TJ.

$\underbrace{f(x_1)=f(x_2)}_{\text{zał}} \Rightarrow \underbrace{x_1=x_2}_{\text{TEZA}}$ } dobre do udowodnienia że jest (R)

$f(x_1)-f(x_2)=0$

$\Updownarrow$

Z: $f(x)=\frac{\sqrt{3}x-2}{4x-3}$ $D: x\in\mathbb{R}\setminus\{\frac{3}{4}\}$ $f(x_1)=f(x_2)$

T: $x_1=x_2$

D: $0=f(x_1)-f(x_2)=\frac{\sqrt{3}x_1-2}{4x_1-3}-\frac{\sqrt{3}x_2-2}{4x_2-3}=\frac{(\sqrt{3}x_1-2)(4x_2-3)}{(4x_1-3)(4x_2-3)}-\frac{(\sqrt{3}x_2-2)(4x_1-3)}{(4x_1-3)(4x_2-3)}=$

$=\frac{4\sqrt{3}x_1x_2-3\sqrt{3}x_1-8x_2+6-(4\sqrt{3}x_1x_2-3\sqrt{3}x_2-8x_1+6)}{(4x_1-3)(4x_2-3)}=$

$\frac{-3\sqrt{3}x_1+8x_1-8x_2+3\sqrt{3}x_2}{M}=\frac{8(x_1-x_2)-3\sqrt{3}(x_1-x_2)}{M}$

$\frac{(x_1-x_2)(8-3\sqrt{3})}{M}=0 \Leftrightarrow x_1-x_2=0 \Leftrightarrow x_1=x_2$

$8-3\sqrt{3}\neq 0$

$0=f(x_1)-f(x_2) \Rightarrow x_1-x_2=0 \Rightarrow f$ jest różnowart.

2: $f(x) = -4\sqrt{x+5} + 9$ $\boxed{x \geq -5}$

$f(x_1) - f(x_2) = 0$

$\boxed{\sqrt{a} - \sqrt{b} = \dfrac{a-b}{\sqrt{a}+\sqrt{b}}}$

T: $x_1 - x_2 = 0$

D: $\overset{0}{\underset{\shortparallel}{}}\, f(x_1) - f(x_2) = -4\sqrt{x_1+5} + 9 + 4\sqrt{x_2+5} - 9 =$

$4\left(\sqrt{x_2+5} - \sqrt{x_1+5}\right) = \dfrac{4\left[(x_2+\cancel{5}) - (x_1+\cancel{5})\right]}{\sqrt{x_2+5} + \sqrt{x_1+5}} = \dfrac{4(x_2-x_1)}{\sqrt{\ } + \sqrt{\ }} \Rightarrow$

$\Rightarrow x_2 - x_1 = 0 \Rightarrow$ f. Różnowartościowa