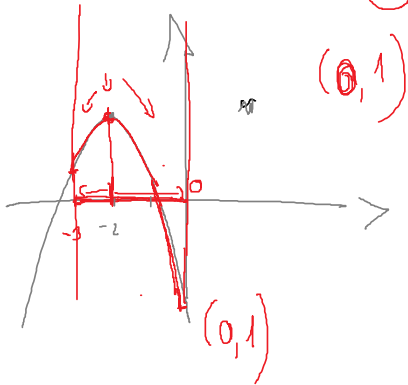


$x \in (-\infty, -2) \Rightarrow f \nearrow$ $x \in (-2, +\infty) \Rightarrow f \searrow$

$x \in (-3, 0)$
 $y_{\max} = 4$
 $y_{\min} = 1$



$p \in (-3, 0) \Rightarrow$

$y_{\max} = 4 = q$

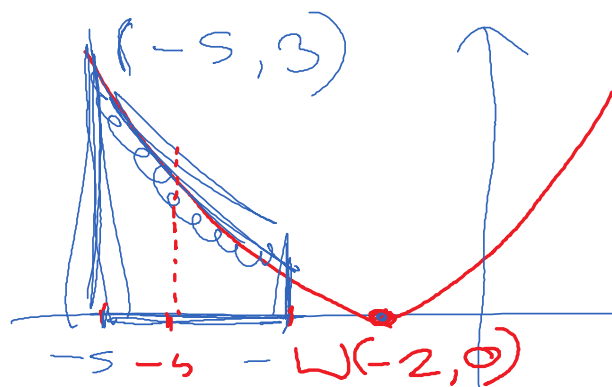
$W(-2, 4) \Rightarrow y = a(x - (-2))^2 + 4$

$(0, 1) \in y = a(x + 2)^2 + 4 \quad a = ?$

$1 = a(0 + 2)^2 + 4$
 $a = -\frac{3}{4}$

$y = -\frac{3}{4}(x + 2)^2 + 4$

$y = -\frac{3}{4}x^2 - 3x + 1$



$$y = \frac{1}{3}(x+2)^2$$

$$y = \frac{1}{3}x^2 + \frac{4}{3}x + \frac{4}{3}$$

$$y = a(x+2)^2 + 0$$

$$a(x-p)^2 + q$$

$\Downarrow$

$$y = a(x+2)^2$$

$$3 = a(-5+2)^2$$

$$3 = a \cdot 9$$

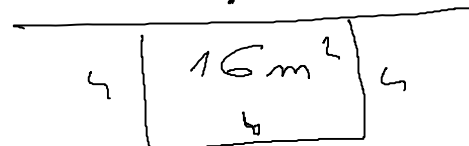
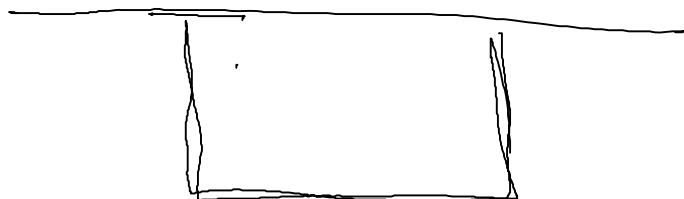
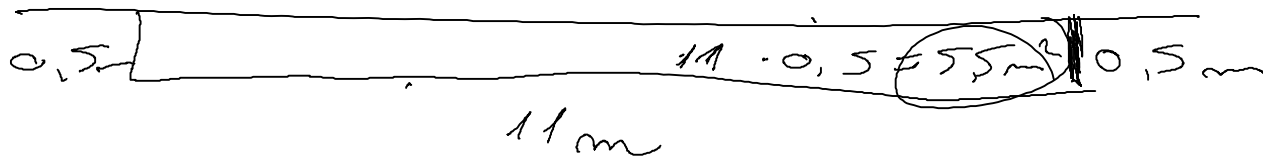
$$a = \frac{1}{3}$$

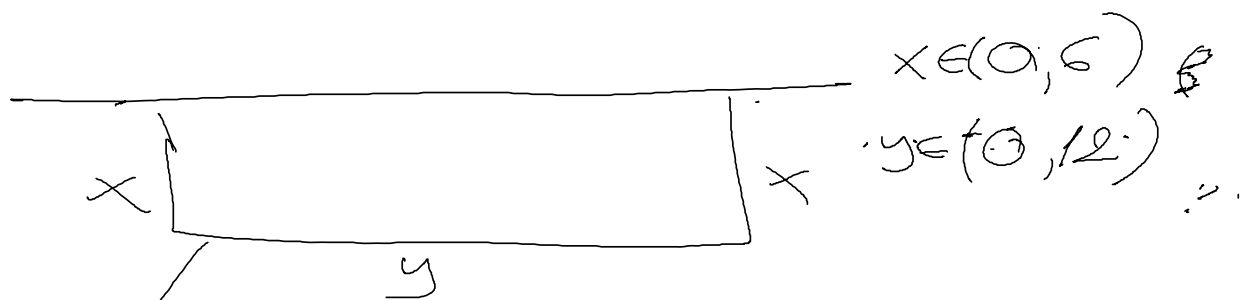
3.105

/

3.102P

12 m sz.



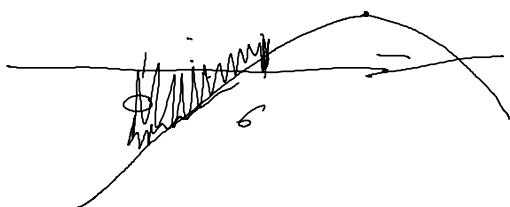


$$2x + y = 12$$

$$y = 12 - 2x$$

P_{max}

$$f(x, y) = x \cdot y = x(12 - 2x) = -2x^2 + 12x$$



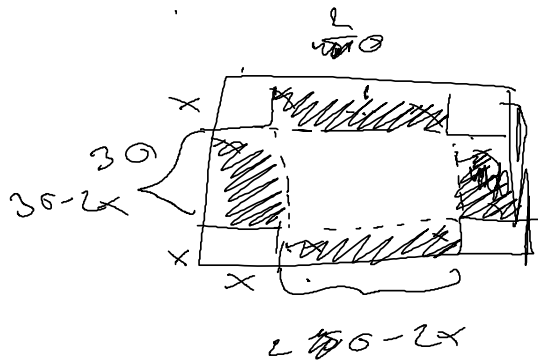
$$x_{\text{W}} = P = \frac{-b}{2a} = \frac{-12}{-4} = 3 \in (0, 6)$$

$$x = 3 \quad y = 12 - 2x = 12 - 6 = 6$$

$$16 \text{ m}^2$$

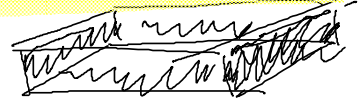
$$18 \text{ m}^2$$

$$P = 3 \cdot 6 = 18$$



$$x > 0 \wedge x < 10$$

$$x \in (0, 10)$$



$$P_k = (30x - 2x^2) \cdot 2x + (20 - 2x) \cdot 2x$$

$$P_k = 60x - 4x^2 + 40x - 4x^2 = -8x^2 + 100x$$

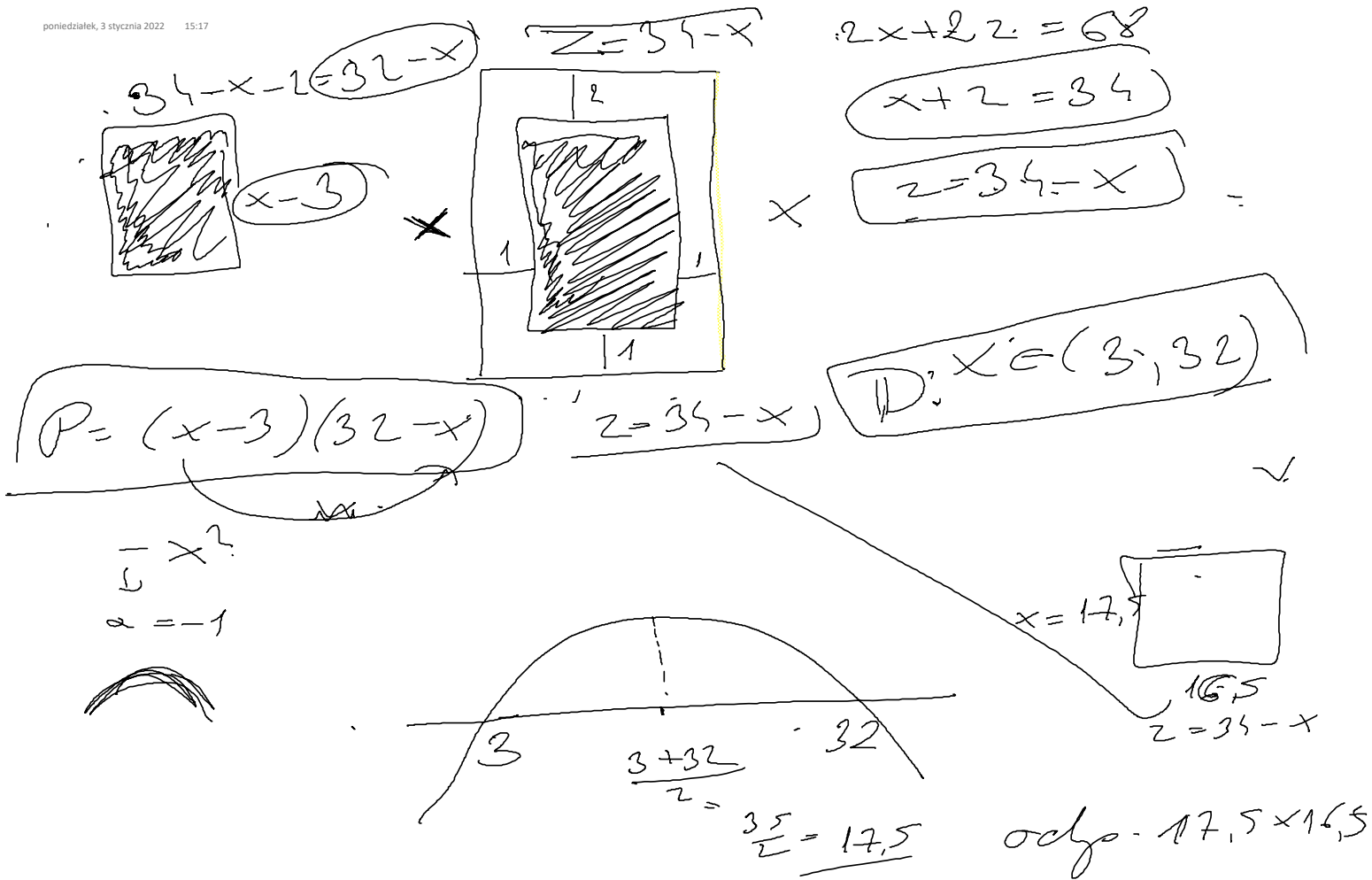
$$P_k = -8x(x - 12,5)$$

0 12,5



$$P_k = -8 \cdot 6,25 \cdot (-6,25)$$

$$P = \frac{2}{8} \cdot \frac{12,5}{2} \cdot \frac{12,5}{2} = \frac{25 \cdot 12,5}{2} = 312,5$$



P 3.107 108 109
R 110 111 112

120 w

k: 1200

s_y: 1800

1800 - 10

$$40 \times 4000 \frac{2}{600} = 245K$$

$$1 \quad 41 \times 590$$

$$2 \quad 42 \times 580$$

$$10 \quad 50 \times 500$$

$$\textcircled{x} \quad (40+x) \cdot (600-10x) = 245K.$$

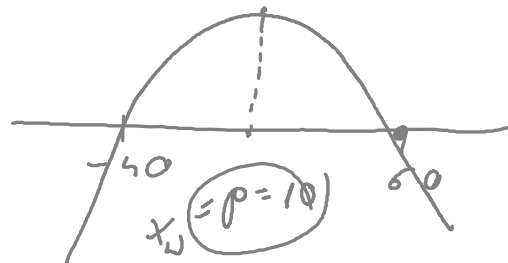
$$2 \quad \frac{1}{2}(x) = (40+x)(-10)(x-60)$$

$$\frac{1}{2}(x) = -10(x+40)(x-60) \quad \text{— potać iloczyn —}$$

$$a(x-x_1)(x-x_2)$$

$$a < 0$$

$$\textcircled{II}: x \in \{0, 1, 2, \dots, 60\}$$



d $x=10$ — zysk jest największy $\Rightarrow$ cena = $1200 + 600 - 10x$

$$\text{cena} = 1200 + 500 = 1700$$

180 lch wylch

$$\underline{Zysk = 180 \cdot 1200}$$

$$z = (180 - 5)(1200 + 40)$$

$$z = (180 - 2 \cdot 5)(1200 + 2 \cdot 40)$$

$$z = (180 - 5 \cdot 10)(1200 + 40 \cdot 10)$$

$$\textcircled{D}: x \in \{0, 1, 2, \dots, 36\}$$

Najmniejszy z nich gdy a w lch $1200 + 3 \cdot 40 = 1320$ i
 $\underline{180 - 15 = 165}$

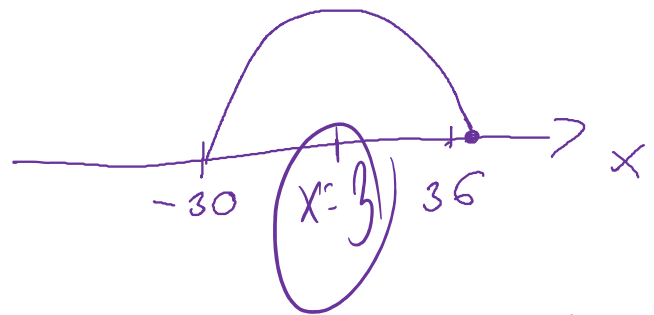
1200 i w lch

$$40(30+x)(-5)(-36+x)$$

$$z(x) = \overset{40}{\uparrow} (1200 + 40x) \overset{-5}{\uparrow} (180 - 5x)$$

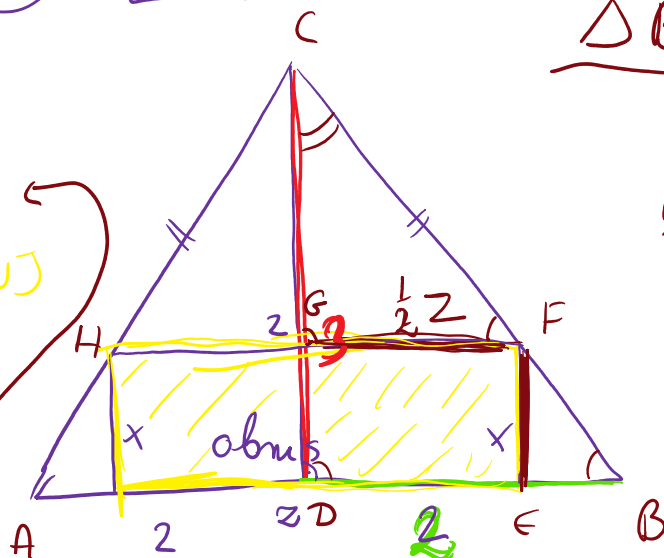
$$z(x) = a(x - x_1)(x - x_2)$$

$$z(x) = -200(x + 30)(x - 36)$$



108 - 105p

$$\triangle BCD \sim \triangle FCG \text{ (kkk)}$$



$$\frac{BC}{FC} = \frac{BD}{FG} = \frac{CD}{CG}$$

$$\frac{2}{\frac{1}{2}z} = \frac{3}{3-x}$$

$$\frac{3}{2}z = 6 - 2x \quad | \cdot \frac{2}{3}$$

$$z = 4 - \frac{4}{3}x$$

$$P(x, z) = x \cdot z$$

dwa niewiadome FUJ

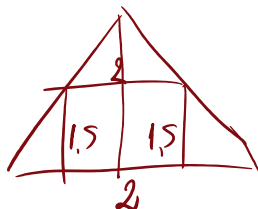
$$P(x) = x \cdot \left(4 - \frac{4}{3}x\right)$$

$$P(x) = \frac{4}{3} \cdot x \cdot (x - 3)$$



$$\textcircled{II} : x \in (0, 3)$$

$$p = x_{\text{m}} = 1,5 = \frac{3}{2} \Rightarrow z = 4 - \frac{4}{3} \cdot \frac{3}{2} = 2$$



Równanie kwadratu

$$ax^2 + bx + c = 0$$

$$a \neq 0$$

1)

3.121 - 3.129
po zostaniu
pod

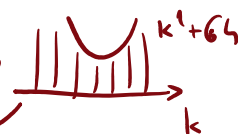
3.255. Dla jakich wartości parametru k , $k \in \mathbb{R}$, suma odwrotności różnych rozwiązań równania $x^2 + kx - 16 = 0$ jest równa -4 ?

$$\begin{aligned} a_x &= 1 \neq 0 \\ b_x &= k \\ c_x &= -16 \end{aligned}$$

$$\begin{aligned} 1) & \begin{cases} a_x \neq 0 \\ \Delta_x > 0 \\ \frac{1}{x_1} + \frac{1}{x_2} = -4 \end{cases} \Leftrightarrow \left\{ \begin{array}{l} k \in \mathbb{R} \\ k \in \mathbb{R} \\ k = -64 \end{array} \right\} \Rightarrow \text{odp } k = -64 \end{aligned}$$

ad 2 $\Delta_x = b^2 - 4ac = k^2 + 64$

$$\Delta_x > 0 \Leftrightarrow k^2 + 64 > 0 \Leftrightarrow k \in \mathbb{R}$$



ad 3 $\frac{1}{x_1} + \frac{1}{x_2} = -4 \quad L = \frac{x_2 + x_1}{x_1 \cdot x_2} = -\frac{\frac{b_x}{a_x}}{\frac{c_x}{a_x}} = -\frac{b_x}{c_x} = \frac{-k}{-16} = \frac{k}{16} = -4$

$$k = -64$$

3.259. Wyznacz wszystkie wartości parametru k , dla których różne rozwiązania x_1, x_2 równania $x^2 - (k-1)x + 1 = 0$ spełniają warunek $\frac{1}{x_1^2} + \frac{1}{x_2^2} \geq 2k^2 - k - 21$.

$$a_x = 1 \neq 0$$

$$b_x = -(k-1)$$

$$c_x = 1$$

$$\begin{cases} 1) a_x \neq 0 \\ 2) \Delta_x > 0 \end{cases}$$

$$3) \frac{1}{x_1^2} + \frac{1}{x_2^2} \geq 2k^2 - k - 21$$

$$\Leftrightarrow \begin{cases} k \in \mathbb{R} \\ k \in (-\infty, -1) \cup (3, +\infty) \\ k \in (-5, 4) \end{cases}$$

$$\Leftrightarrow \text{odp. } k \in (-5, -1) \cup (3, 4)$$

$$\Delta_x = (k-1)^2 - 4 = k^2 - 2k + 1 - 4 = k^2 - 2k - 3 = \Delta_x$$

$$\Delta_x > 0 \Leftrightarrow k^2 - 2k - 3 > 0$$

Nierówność kwadratowa.

1. Posprzątać
2. Obliczyć miejsca zerowe trójmianu kwadratowego ax^2+bx+c
3. ZAWSZE RYSUJEMY WYKRES PARABOLI!!!
 Uśmiech/smutas są miejsca zerowe/brak miejsc zerowych
4. Odczytujemy, która część paraboli jest nad/pod osią

$$\Delta_k = (-2)^2 + 4 \cdot 3 = 16 = 4^2$$

$$k_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{2 - 4}{2} = -1 \quad k_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{2 + 4}{2} = 3$$

$$(k+1)(k-3) > 0$$

$$k \in (-\infty, -1) \cup (3, +\infty)$$



ad 3) $\frac{1}{x_1^2} + \frac{1}{x_2^2} \geq 2k^2 - k - 21$

$$L = \frac{x_1^2 + x_2^2}{(x_1 \cdot x_2)^2} = \frac{(x_1 + x_2)^2 - 2x_1x_2}{(\frac{c}{a})^2} =$$

$$\frac{\left(\left(-\frac{b}{a}\right)^2 - 2 \cdot \frac{c}{a}\right) \cdot a^2}{\left(\frac{c}{a^2}\right) \cdot a^2} = \frac{b^2 - 2ac}{c^2} = \frac{(k-1)^2 - 2 \cdot 1 \cdot 1}{1^2} =$$

$$1^2 - 0 \cdot 1 + 1 - 1 = \sqrt{1^2 - 0 \cdot k - 1} = L$$

$$k^2 - 2k + 1 - 2 = \boxed{k^2 - 2k - 1} = L$$

$$\boxed{k^2 - 2k - 1} \geq \boxed{2k^2 - k - 21}$$

N.K.W.

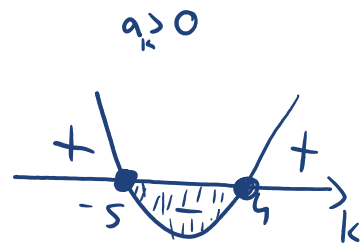
$$\boxed{k^2 + k - 20 \leq 0}$$

k_1 k_2

$$\Delta_k = b^2 - 4ac = 1 + 80 = 81 = 9^2$$

$$k_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-1 - 9}{2} = -5$$

$$k_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-1 + 9}{2} = 4$$



$k \in \langle -5; 4 \rangle$ ↑

$\Delta \geq 0$ bo mogą być $x_1 = x_2 \Leftrightarrow \Delta = 0$
 ale nie ma dwóch różnych rozwiązań

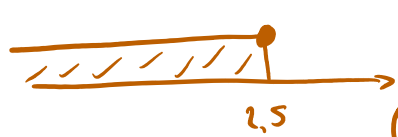
$$\begin{aligned} a_x &= 1 \neq 0 \\ b_x &= 2(p-1) \\ c_x &= p^2 - 4 \end{aligned}$$

3.257. Dla jakich wartości parametru p , $p \in \mathbb{R}$, równanie $x^2 + 2(p-1)x + p^2 - 4 = 0$ ma dwa rozwiązania, których suma kwadratów jest mniejsza od 12?

$$\begin{aligned} 1 & \begin{cases} a_x \neq 0 \\ \Delta_x \geq 0 \\ x_1^2 + x_2^2 < 12 \end{cases} \Leftrightarrow \begin{cases} p \in \mathbb{R} \\ p \in (-\infty; 2,5) \\ p \in (0,4) \end{cases} \end{aligned}$$

odp $\Leftrightarrow p \in (0; 2,5)$

ad 2) $\Delta_x \geq 0$ $\Delta_x = b^2 - 4ac = [2(p-1)]^2 - 4 \cdot 1 \cdot (p^2 - 4) =$
 $4(p^2 - 2p + 1) - 4p^2 + 16 = -8p + 4 + 16 = -8p + 20$

$\Delta_x \geq 0 \Leftrightarrow -8p + 20 \geq 0$ NIERÓWNOŚĆ LINIOWA $\Rightarrow -8p \geq -20 \quad | :(-8) < 0$
 $p \leq 2,5$

 $p \in (-\infty; 2,5)$

ad 3) $x_1^2 + x_2^2 < 12$ $L = x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = \left(-\frac{b_x}{a_x}\right)^2 - 2 \frac{c_x}{a_x} =$

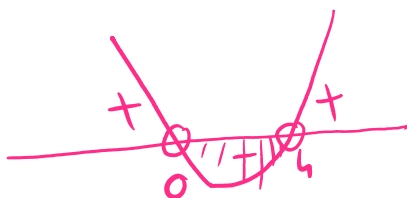
$$= \frac{b^2}{a^2} - 2 \frac{c}{a} = \frac{(2(p-1))^2}{1} - \frac{2(p^2 - 4)}{1} = 4(p^2 - 2p + 1) - 2p^2 + 8 =$$

$$= 2p^2 - 8p + 12 = L$$

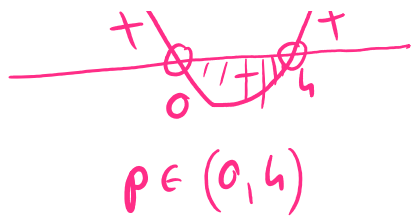
$$2p^2 - 8p + 12 < 12 \quad | :2$$

$$p^2 - 4p < 0$$

$$p(p-4) < 0$$



$$0 < p < 4$$



$$0 \leq \frac{1}{4}$$