

$$\begin{aligned} \operatorname{tg}\left(\frac{\alpha}{2} - \frac{\pi}{8}\right) &= \operatorname{tg}\left(\frac{1}{2}\left(\alpha - \frac{\pi}{4}\right)\right) \\ \operatorname{tg}\frac{1}{2}\left(\alpha - \frac{\pi}{4}\right) &= \frac{1 - \cos\left(\alpha - \frac{\pi}{4}\right)}{\sin\left(\alpha - \frac{\pi}{4}\right)} \\ \operatorname{tg} 2x &= \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x} \quad x = \frac{2y}{1-y^2} \\ \cos 2x &= 2 \cos^2 x - 1 \quad \cos x = \pm \sqrt{\frac{1}{2} + \frac{1}{2} \cos 2x} \\ \cos 2x &= 1 - 2 \sin^2 x \quad \sin x = \pm \sqrt{\frac{1}{2} - \frac{1}{2} \cos 2x} \\ \operatorname{tg} x &= \pm \sqrt{\frac{\frac{1}{2} - \frac{1}{2} \cos 2x}{\frac{1}{2} + \frac{1}{2} \cos 2x}} = \pm \sqrt{\frac{(1 - \cos 2x)^2}{\sin^2 2x}} = \pm \frac{1 - \cos 2x}{\sin 2x} \\ \operatorname{tg} \frac{1}{2} x &= \frac{1 - \cos x}{\sin x} \quad \begin{array}{l} \text{I } x \in (0, \pi) \Rightarrow \frac{1}{2} x \in (0, \frac{\pi}{2}) \\ \text{II } x \in (\pi, 2\pi) \Rightarrow \frac{1}{2} x \in (\frac{\pi}{2}, \pi) \end{array} \\ \operatorname{tg} \frac{1}{2} x &= \frac{-1 \pm \sqrt{\operatorname{tg}^2 x + 1}}{\operatorname{tg} x} = \frac{-\cos x \pm 1}{\sin x} = \pm \frac{1 - \cos x}{\sin x} \end{aligned}$$

$$\begin{aligned} \sin x + \log(1 + 2^{\sin x}) &= \sin x \log 5 + \log 6 \\ \log 10 - \sin x - \sin x \log 5 &= \log 6 - \log(1 + 2^{\sin x}) \quad (1) \\ \sin x \cdot \log \frac{1}{5} &= \log \left(\frac{1 + 2^{\sin x}}{6}\right) \\ \log \frac{1}{5}^{\sin x} &= \log \frac{1 + 2^{\sin x}}{6} \\ \left(\frac{1}{5}\right)^{\sin x} &= \frac{1 + 2^{\sin x}}{6} \\ 6 \cdot 2^{-\sin x} &= 1 + 2^{\sin x} \\ 6 \cdot \frac{1}{5} &= 1 + 5 \\ 6 &= 1 + 5 \\ 1^2 + 5 - 6 &= 0 \\ (t + 3)(t - 2) &= 0 \\ t &= -3 \vee 2 \\ 2^{\sin x} &\neq -3 \\ 2^{\sin x} &= 2 \Rightarrow \sin x = 1 \Rightarrow x = \left(\frac{\pi}{2} + 2k\pi\right) \end{aligned}$$

$$\begin{aligned} \frac{1 - \cos\left(2 - \frac{\pi}{4}\right)}{\sin\left(2 - \frac{\pi}{4}\right)} &= \frac{1 - (\cos 2 \cos \frac{\pi}{4} + \sin 2 \sin \frac{\pi}{4})}{\sin 2 \cos \frac{\pi}{4} - \sin \frac{\pi}{4} \cos 2} \\ &= \frac{1 - \frac{1}{\sqrt{2}}(\cos 2 - \sin 2)}{\frac{1}{\sqrt{2}}(\sin 2 - \cos 2)} = \frac{\sqrt{2} - \cos 2 + \sin 2}{\sin 2 - \cos 2} \end{aligned}$$

$$\begin{aligned} \sqrt{2} - \cos \alpha - \sin \alpha &= -(\cos \alpha - \cos \frac{\pi}{4}) - (\sin \alpha - \sin \frac{\pi}{4}) \\ 2 \sin\left(\frac{\alpha}{2} + \frac{\pi}{8}\right) \sin\left(\frac{\alpha}{2} - \frac{\pi}{8}\right) &- 2 \cos\left(\frac{\alpha}{2} + \frac{\pi}{8}\right) \sin\left(\frac{\alpha}{2} - \frac{\pi}{8}\right) \\ 2 \sin\left(\frac{\alpha}{2} - \frac{\pi}{8}\right) \left(\sin\left(\frac{\alpha}{2} + \frac{\pi}{8}\right) - \cos\left(\frac{\alpha}{2} + \frac{\pi}{8}\right)\right) \\ 2 \sin\left(\frac{\alpha}{2} - \frac{\pi}{8}\right) \left(\sin\left(\frac{\alpha}{2} + \frac{\pi}{8}\right) - \cos\left(\frac{\alpha}{2} + \frac{\pi}{8}\right)\right) \\ 2 \sin^2\left(\frac{\alpha}{2} - \frac{\pi}{8}\right) \end{aligned}$$

$$\begin{aligned} \sin^8 x + \cos^6 x &= 1 \quad x = \frac{\pi}{2} \\ \sin^8 x &= 1 - \cos^6 x = (1 - \cos^2 x)(1 + \cos^2 x + \cos^4 x) \\ a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \\ \sin^8 x &= \sin^2 x (\cos^4 x + \cos^2 x + 1) \\ \sin^2 x (-\sin^6 x + 1 + \cos^4 x + \cos^2 x) &= 0 \\ (1 - \sin^2 x) &= 0 \end{aligned}$$

$$\sin^8 x + \cos^8 x = \sin^2 x + \cos^2 x$$

$$\sin^2 x (\sin^6 x - 1) = -\cos^2 x (\cos^6 x - 1)$$

$$\sin^2 x (\sin^2 x - 1)(\sin^4 x + 1) = -(1 - \sin^2 x)(\cos^2 x - 1)(\cos^4 x + 1)$$

$$\sin^2 x (\sin^2 x - 1)((\sin^2 x + 1)^2 - \sin^2 x) = (\sin^2 x - 1) \cdot \sin^2 x \cdot (\sin^2 x - 2)$$

$$\sin^2 x (\sin^2 x - 1)(\sin^4 x + 2\sin^2 x + 1 - \sin^2 x - \sin^2 x - 2) = 0 \quad k \in \mathbb{Z}$$

$$\sin^2 x (\sin^2 x - 1)(\sin^4 x + 3) = 0 \Leftrightarrow \sin^2 x \in \{-1, 0, 1\} \Leftrightarrow x = \frac{k\pi}{2}$$

$$(1 - \cos^2 x)^4 + \cos^6 x = 1$$

$$\cos^8 x - 4\cos^6 x + 6\cos^4 x - 4\cos^2 x + 1 + \cos^6 x = 1$$

$$\cos^2 x (\cos^6 x - 3\cos^4 x + 6\cos^2 x - 4) = 0$$

$$\cos^2 x (\cos^2 x - 1)(\cos^4 x - 2\cos^2 x + 1) = 0$$

$$\log_{\sin x} \cos x + \log_{\cos x} \sin x = 2$$

$\sin x = a$
 $\cos x = b$
 $a^n = b$
 $b^m = a$
 $a^{mn} = a \Leftrightarrow a = 1 \vee m \cdot n = 1$

$\begin{cases} m+n=2 \\ m \cdot n=1 \end{cases}$
 $m=2-n$
 $(2-n) \cdot n = 1$
 $-n^2 + 2n - 1 = 0 \quad | \cdot (-1)$
 $(n-1)^2 = 0$
 $n=1 \Rightarrow m=1$
 $\sin x = \cos x = \frac{\sqrt{2}}{2} + 2k\pi$

$\log_{\sin x} \cos x = \frac{\log \sin x \cos x}{\log \sin x}$
 $\log_{\cos x} \sin x = \frac{\log \sin x \cos x}{\log \cos x}$
 $\frac{\log \sin x \cos x}{\log \sin x} + \frac{\log \sin x \cos x}{\log \cos x} = 2$
 $\log \sin x \cos x > 0$
 $x \in (2k\pi, \frac{\pi}{2} + 2k\pi) \cup (\frac{3\pi}{2} + 2k\pi, 2\pi + 2k\pi)$

$$\log_{\sin x} \cos x + \log_{\cos x} \sin x = 2$$

$x = \frac{\pi}{4} + k\pi$

1) I WERSZUK. $x \in \mathbb{C}$

2) $p + \frac{1}{p} = 2 \quad \log_a b \cdot \log_b a = 1$

3) $p > 0 \Rightarrow p + \frac{1}{p} \geq 2 \quad | \quad p + \frac{1}{p} = 2 \Leftrightarrow p = 1$

4) $p = 1 = \log_{\sin x} \cos x \Leftrightarrow \sin x = \cos x \quad \mathbb{C}$

$\sin x^{\log_{\sin x} \cos x} = \sin x$