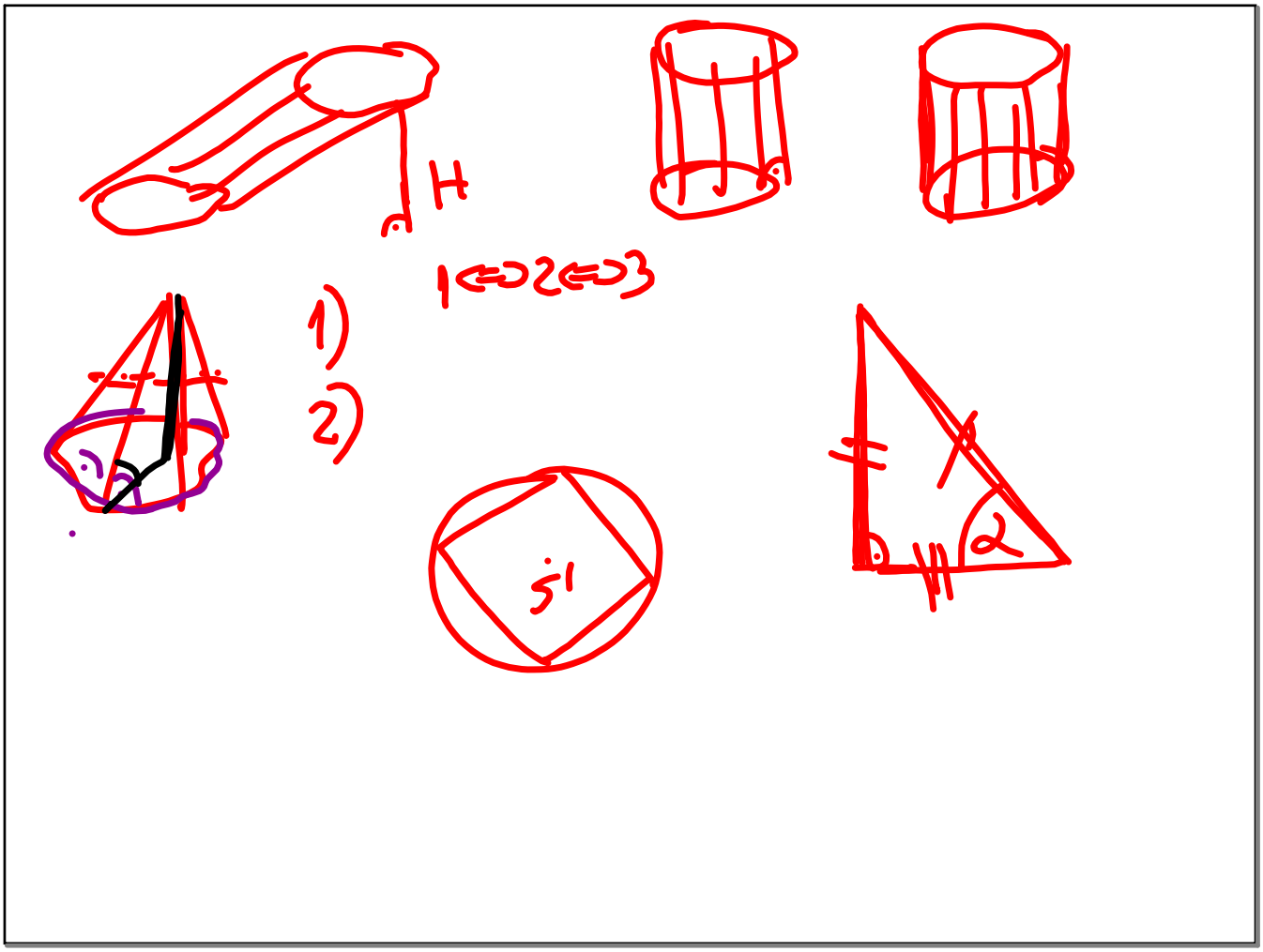
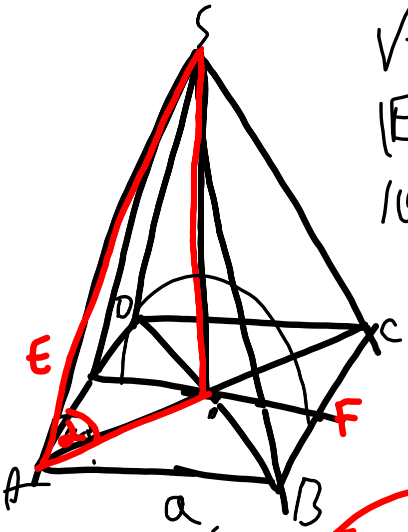

H
1⇐2⇐3
1)
2)
5
1
2



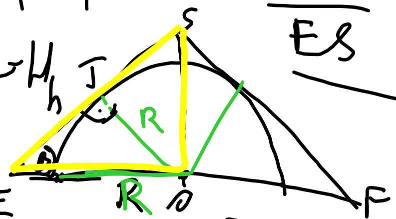
$V=\frac{1}{3}a^2H$

$|ES|=h$

$|OS|=H$

$\triangle EOT \sim \triangle ESO$

$\frac{EO}{ES}=\frac{OT}{SO}=\frac{ET}{EO}$

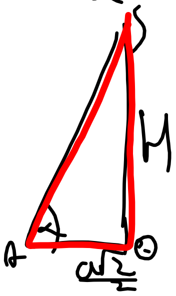


$\frac{a}{2h}=\frac{R}{H}$

$h=\sqrt{H^2+\frac{1}{4}a^2}$

$|EF|=a$

$\frac{a^2}{4H^2+a^2}=\frac{R^2}{H^2}$



$tg\alpha=\frac{H\sqrt{2}}{a}$

$\frac{a\,tg\alpha}{\sqrt{2}}=H$

$\frac{a^2}{2a^2tg^2\alpha+a^2}=\frac{2R^2}{a^2tg^2\alpha}$

$\frac{1}{2tg^2\alpha+1}=\frac{2R^2}{a^2tg^2\alpha}$

$a^2=\frac{4R^2tg^2\alpha+2R^2}{tg^2\alpha}$

$V=\frac{1}{3}\frac{2R^2(1+2tg^2\alpha)}{tg^2\alpha}\cdot\frac{\sqrt{\frac{2R^2(1+2tg^2\alpha)}{tg^2\alpha}}\cdot tg\alpha}{\sqrt{2}}$

$=\frac{2}{3}R^3\frac{(1+2tg^2\alpha)}{tg^2\alpha}\cdot\sqrt{1+2tg^2\alpha}$

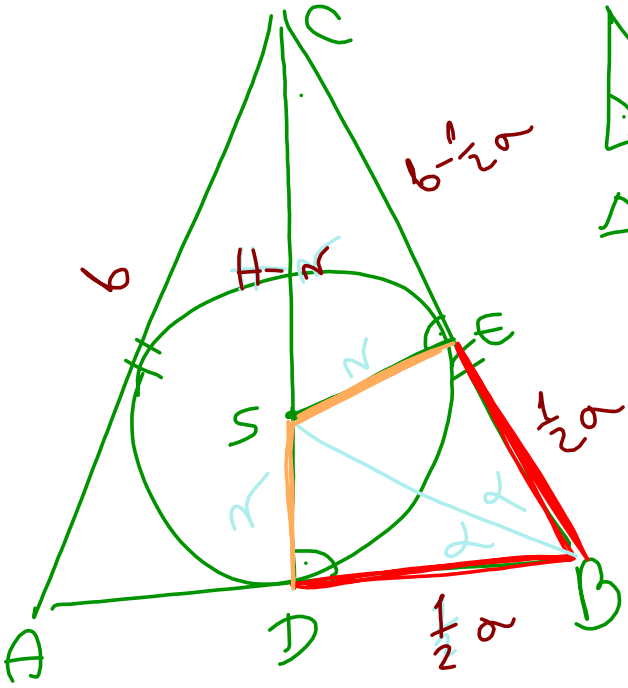
$V=\frac{2}{3}R^3\frac{(2-\cos^2\alpha)^{\frac{3}{2}}}{\sin^2\alpha\cos\alpha}$

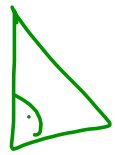
T: $\frac{1+2tg^2\alpha}{tg^2\alpha}\cdot\sqrt{1+2tg^2\alpha}\overset{?}{=}\frac{(2-\cos^2\alpha)^{\frac{3}{2}}}{\sin^2\alpha\cos\alpha}$

$\frac{\cos^2\alpha+2\sin^2\alpha}{\sin^2\alpha}\cdot\sqrt{\frac{\cos^2\alpha+2\sin^2\alpha}{\cos^2\alpha}}\overset{?}{=}\frac{(2-\cos^2\alpha)^{\frac{3}{2}}}{\sin^2\alpha\cos\alpha}$

$(2-\cos^2\alpha)\sqrt{(2-\cos^2\alpha)}=(2-\cos^2\alpha)^{\frac{3}{2}}=P$

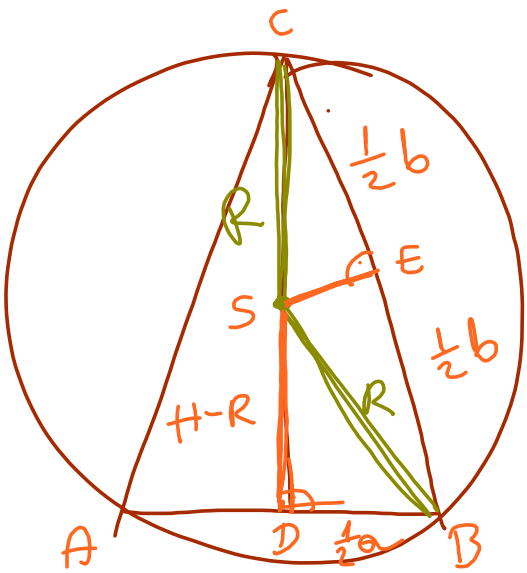
q.e.d.





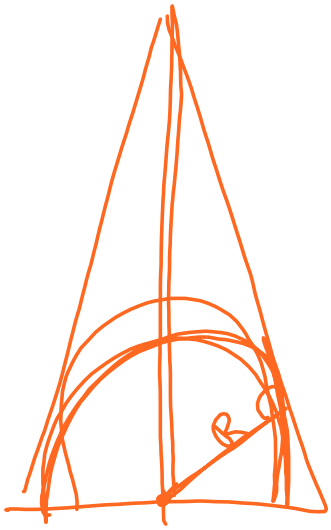
CDB $a^2+b^2=c^2$

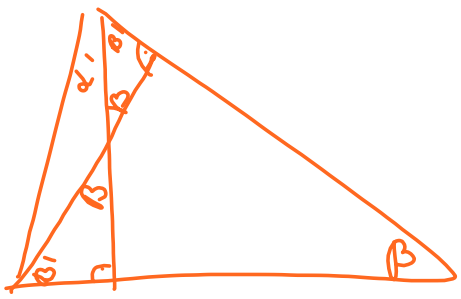
$\triangle SEC \sim \triangle BDC$

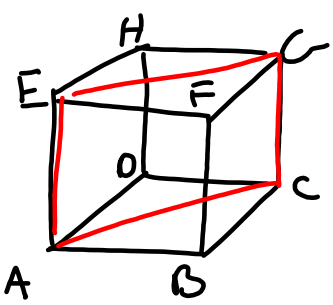


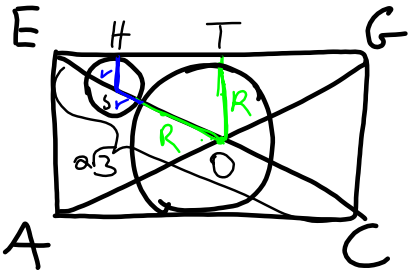
$\triangle DBC \sim \triangle ESC$

PIETIA GOLAS







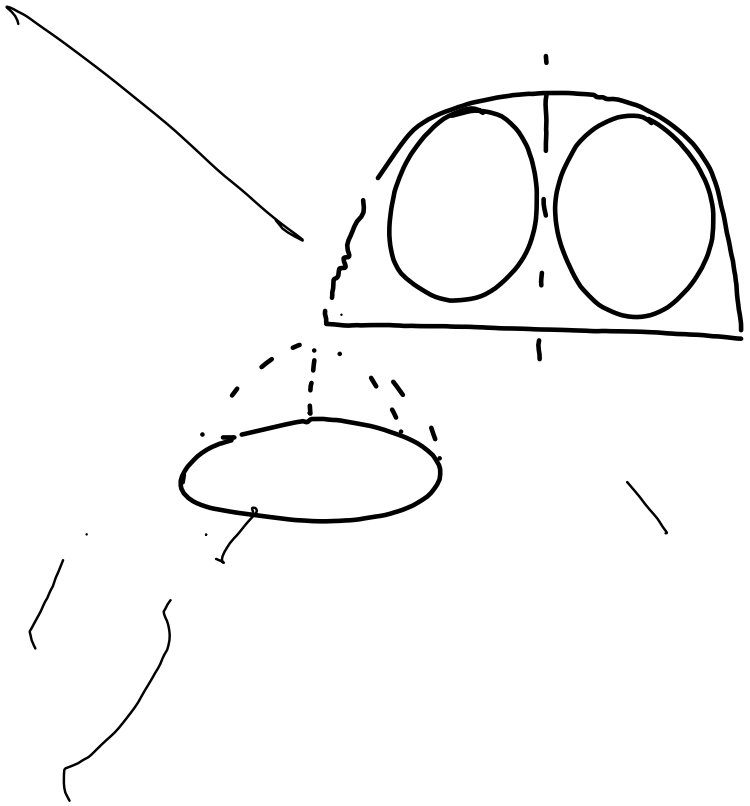


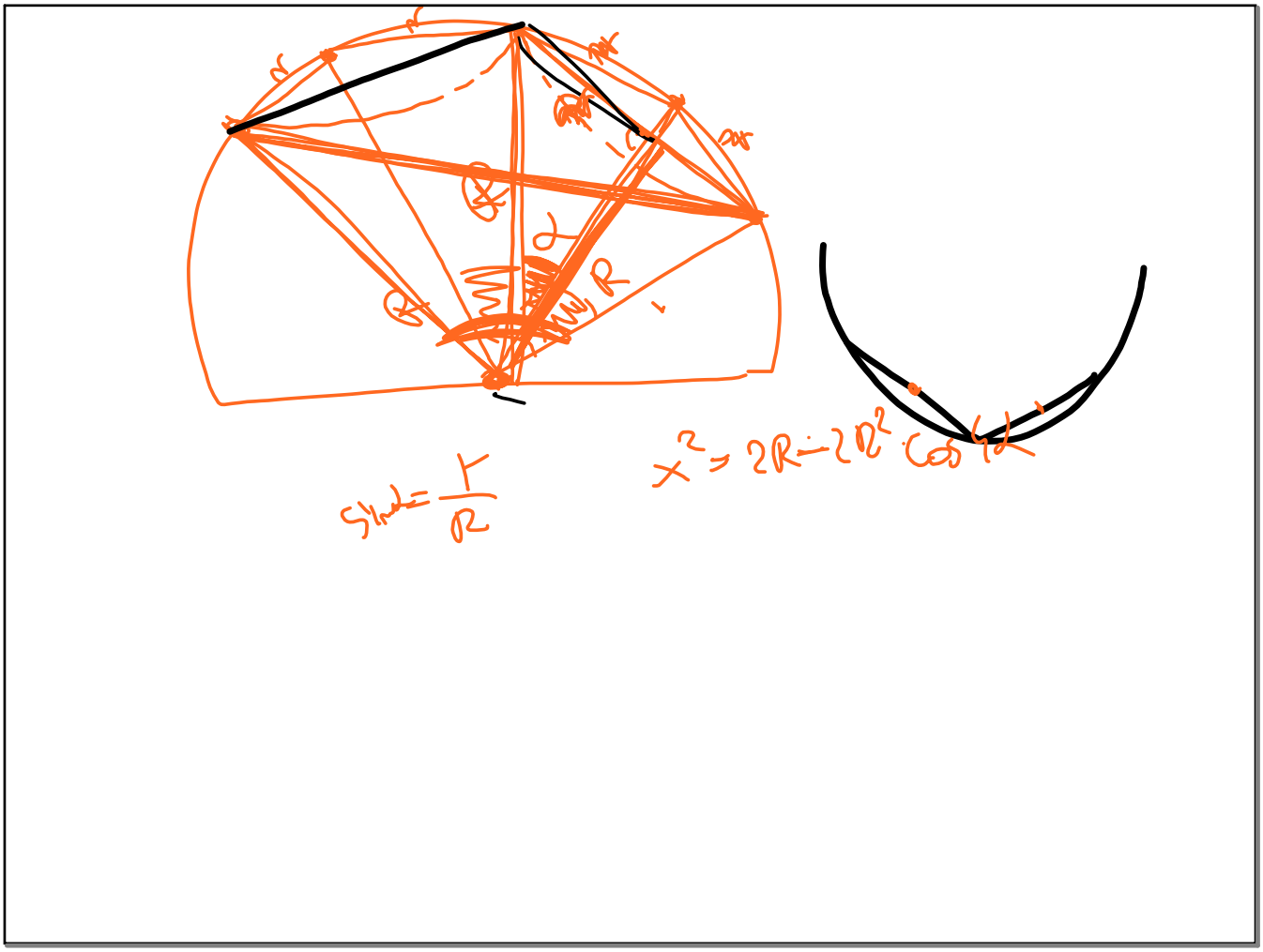
$\triangle EHS \sim \triangle ETO$

$\frac{r}{R} = \frac{ES}{EO}$ $\quad EO = \frac{a\sqrt{3}}{2} = R\sqrt{3}$

$\frac{r}{R} = \frac{R\sqrt{3} - R - r}{R\sqrt{3}}$ $\quad R = \frac{a}{2}$

$\frac{r}{R} = 2 + \sqrt{3}$




R
R
α
R
$\sin\alpha = \frac{r}{R}$
$x^2 = 2R - 2R^2 \cdot \cos\alpha$