

$$V_0 = \frac{1}{3} P_p \cdot H$$

$$V_s = \frac{1}{3} \pi R^2 \cdot H \leftarrow \text{MAX}$$

$$V(R, H) = \frac{1}{3} \pi R^2 \cdot H$$

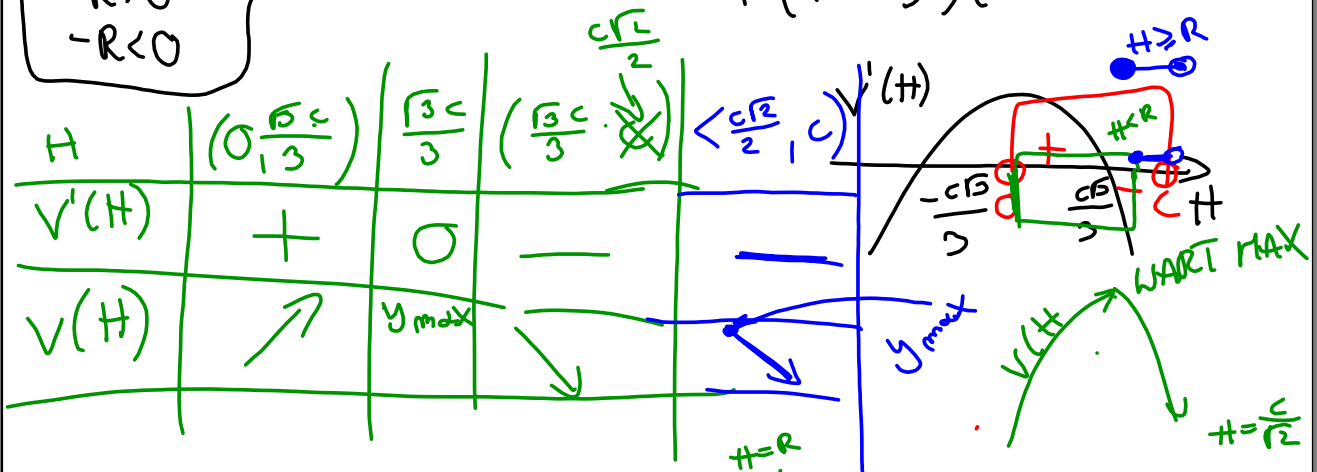
$$\text{I } H < R$$

$$V(H) = \frac{1}{3} \pi (c^2 - H^2) \cdot H = -\frac{1}{3} \pi H^3 + \frac{1}{3} \pi c^2 H$$

$$\begin{aligned} V'(H) &= -\pi H^2 + \frac{1}{3} \pi c^2 = \\ &= -\pi \left(H^2 - \frac{1}{3} c^2 \right) = \\ &= -\pi \left(H - \frac{c\sqrt{3}}{3} \right) \left(H + \frac{c\sqrt{3}}{3} \right) \end{aligned}$$

$$H = c - R < c - 0 = c$$

$$\begin{aligned} R &> 0 \\ -R &< 0 \end{aligned}$$

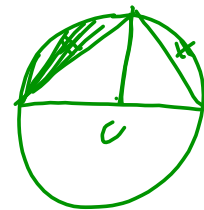
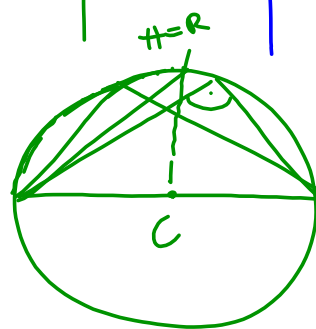
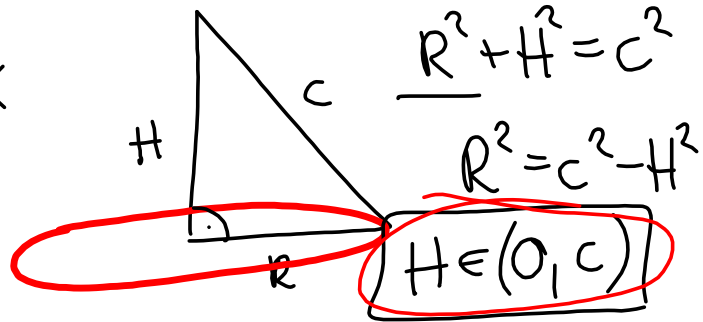


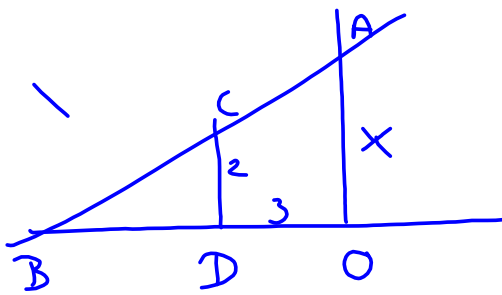
$$\begin{aligned} H &< R \\ H &\in (0, \frac{c}{\sqrt{2}}) \\ (0, \frac{c\sqrt{2}}{2}) \end{aligned}$$

$$\frac{c}{\sqrt{2}} > \frac{c}{\sqrt{3}}$$

$$H \geq R \Rightarrow V_{\max} \text{ dla } H = R = \frac{c\sqrt{2}}{2}$$

$$D: c > 0$$




 $L(x)$

$$\mathbb{D}_L: x \in (2, +\infty)$$

$$L(x) = \sqrt{x^2 + \frac{9x^2}{(x-2)^2}}$$

$$y = \sqrt{x} \nearrow$$

$$f'(x) = \left(x^2 + \frac{9x^2}{(x-2)^2} \right)' = 2x +$$

$$\left(\left(\frac{3x}{x-2} \right)^2 \right)' = 2 \cdot \frac{3x}{x-2} \cdot \left(\frac{3x}{x-2} \right)' =$$

$$= 2 \cdot \frac{3x}{x-2} \cdot \frac{3 \cdot (x-2) - 3x}{(x-2)^2} = \frac{6x \cdot (-6)}{(x-2)^3} = \frac{-36x}{(x-2)^3}$$

$$f'(x) = \underbrace{2x + \frac{-36x}{(x-2)^3}}_{\frac{2x((x-2)^3 - 18)}{(x-2)^3}} = 2x \left(1 - \frac{18}{(x-2)^3} \right)$$

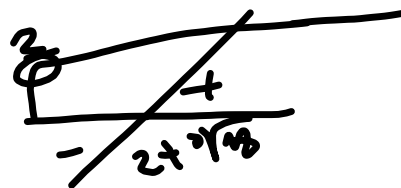
$$f'(x) = 0 \Leftrightarrow x = 0 \vee (x-2)^3 - 18 = 0$$

$$x-2 = \sqrt[3]{18}$$

$$x = \sqrt[3]{18} + 2$$

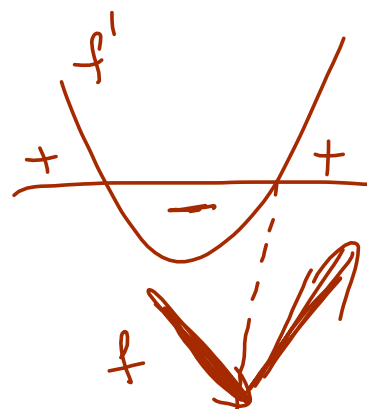
$$\begin{aligned}
 \left(x \sqrt{1 + \frac{9}{(x-2)^2}} \right)' &= \sqrt{1 + \frac{9}{(x-2)^2}} + x \cdot \frac{9 \cdot (-2) \cdot \frac{1}{(x-2)^3}}{2 \sqrt{1 + \frac{9}{(x-2)^2}}} = \\
 &= \frac{1 + \frac{9}{(x-2)^2} - \frac{9x}{(x-2)^3}}{\sqrt{\dots}} = \frac{(x-2)^3 + 9(x-2) - 9x}{\sqrt{\dots} (x-2)^2} = \\
 &= \frac{(x-2)^3 - 18}{\sqrt{\dots} (x-2)^2 (x-2)} \\
 &\quad > 0, \text{ bo } \text{ID: } x > 2
 \end{aligned}$$

$$\begin{aligned}
 (x-2)^3 - 18 &= 0 \\
 x &= \sqrt[3]{18} + 2
 \end{aligned}$$



x	$(2, \sqrt[3]{18} + 2)$	x_0	$(x_0 + \infty)$
$L'(x)$	$-$	0	$+$
$L(x)$	$\searrow$	$y_{\min}$	$\nearrow$

$$\left(9 \cdot (x-2)^{-2} \right)' = 9 \cdot (-2) \cdot (x-2)^{-2-1} \cdot (x-2)'$$



$$A = \{(x, y): \log_y(\underline{8x + y - 2 - x^2}) \geq \log_y(\underline{8 - x^2 + 8x - 2y - y^2})\}$$

$$y \in (0, +\infty) \setminus \{1\}$$

$$\underline{8x - x^2 - 2 + y} > 0 \quad \text{II}$$

$$8x - x^2 + 8 - 2y - y^2 > 0 \quad \text{III}$$

$$(y \in (0, 1) \wedge 8x + y - 2 - x^2 \leq 8 - x^2 + 8x - 2y - y^2) \quad \text{IV}$$

$$(y \in (1, +\infty) \wedge 8x + y - 2 - x^2 \geq 8 - x^2 + 8x - 2y - y^2) \quad \text{V}$$

$$\text{II: } \underline{y > x^2 - 8x + 2}$$

$$y > (x-4)^2 - 14$$

$$\text{III: } \underline{x^2 + y^2 - 8x + 2y - 8 < 0}$$

$$(x-4)^2 - 16 + (y+1)^2 - 1 - 8 < 0$$

$$(x-4)^2 + (y+1)^2 < 5^2$$

$$S(4, -1); r=5$$

$$\text{IV: } y - 2 \leq 8 - 2y - y^2$$

$$y^2 + 3y - 10 \leq 0$$

$$(y+5)(y-2) \leq 0$$

$$y \in \boxed{[-5, 2]} \quad \text{skreślić}$$

$$y \in (0, 1)$$

$$\text{V: } (y+5)(y-2) \geq 0 \wedge y > 1$$

$$y \in [2, +\infty)$$

